Culturally Responsive Mathematics Teaching Practice Guide

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This practice guide is intended to help middle school mathematics educators plan and execute culturally responsive lessons by incorporating practices in three domains: engaged student and community funds of knowledge, interdisciplinary connections, and empowered mathematical inquiry and decision making. This guide was developed as part of the multiyear, mixed-methods Analysis of Middle School Math Systems (AMS) study funded by the Bill & Melinda Gates Foundation. The study aimed to understand the extent to which teachers in four urban school districts plan and execute standards-based, rigorous, and culturally responsive mathematics lessons while using one of six different middle school mathematics curricula.

Introduction

Research has found that culturally responsive teaching supports students who are Black, Latino, English learners, or experiencing poverty—and students who combine these identities and contexts—to have a better classroom experience (Cabrera et al., 2014; Dee & Penner, 2017). Use of culturally responsive practices to improve the mathematics achievement of students from historically marginalized groups is of continued importance; a recent report by the National Assessment of Educational Progress found that, while mathematics scores among 13-year-olds declined for most student groups in the period from 2020-2023, the racial achievement gap between Black and White students widened from 35 points in 2020 to 42 points in 2023 (National Assessment of Educational Progress, 2023).

The AMS study found that most teachers reported receiving professional learning in culturally responsive practices and frequently adapting their instruction to incorporate these practices. Classroom observations however, found limited use of culturally responsive practices in these teachers' instruction. This practice guide aims to bridge the gap between professional learning and implementation by providing tools to help teachers plan and execute culturally responsive mathematics lessons by incorporating practices in three research-based domains: engaged student and community funds of knowledge, interdisciplinary connections, and empowered mathematical inquiry and decision making. **Engaged student and community funds of knowledge** helps students bring their lived experience and intuitive knowledge to the instructional setting as assets for individual and collective learning (Aguirre & del Rosario Zavala, 2013; Jones, 2015; Turner et al., 2012). **Interdisciplinary connections** draws on connections from other content areas and domains of study to advance students' mathematical understanding of a mathematical fact, concept or procedure beyond the lesson (Aguirre & del Rosario Zavala, 2013; Jones, 2015; Turner et al., 2012). **Empowered mathematical inquiry and decision making** engages students in posing questions about societal challenges relevant to them and in tasks that explore and critique them using math as a tool (Aguirre & del Rosario Zavala, 2013; Jones, 2015; Turner et al., 2012). For more information on the link between culturally responsive practices and students' mathematics achievement, as well as the AMS study, please see Appendix A.

Mathematica
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Who are the intended users of this guide

This guide is intended to help middle school math educators, including teachers, coaches, and instructional leaders who are responsible for planning and executing lessons. Although this guide was developed using middle school math examples, the culturally responsive domains are appropriate for use in other subject areas and at other grade levels.

How to use this guide?

There are five main sections of this guide:

- **Section A** provides essential pre-reading and ongoing work to support you as you become a culturally responsive practitioner.
- **Section B** provides a set of tools to support you to incorporate engaged student and community funds of knowledge in your instruction.
- **Section C** provides a set of tools to support you to incorporate interdisciplinary connections in your instruction.
- **Section D** provides a set of tools to support you to incorporate empowered mathematical inquiry and decision making in your instruction.
- **Section E** presents potential misappropriations of culturally responsive teaching practices that teachers should reflect on as part of their culturally responsive praxis..

Contents

Exhibit 1. Tools for each domain

Tool	Purpose
Practices and behaviors of the domain	Descriptions of the different practices and behaviors of the domain, as well as counterexamples that do not illustrate the domain.
Key questions	Big-picture questions to support your learning about the domain and catalyze your thinking about opportunities in upcoming lessons or units to incorporate the domain.
What it looks like	Descriptions of the practices and behaviors of the domain at three levels, representing a progression of instruction from teacher- to student-led activities.
Example lesson	An example lesson incorporating the practices and behaviors of this domain. The example lessons are based on abridged versions of lessons from publicly available curricula, across a range of grade levels and mathematical domains (such as expressions and equations, the number system, statistics and probability, ratios and proportional relationships, and geometry).
Reflection and planning questions	Questions to support reflection on your current incorporation of the practices and behaviors of the domain into your own classroom and to help you make concrete plans for incorporating the domain into an upcoming lesson.

A. Essential readings

Culturally responsive teaching involves more than just the one-time act of revising lessons or using a culturally responsive curriculum. According to Ladson-Billings (1995), a culturally responsive teacher is one for whom "knowledge is continuously recreated, recycled, and shared by the teachers and the students" (p. 163). The teacher's pedagogy will therefore "change and evolve to meet the needs of each generation of students"(Ladson-Billings, 2004, pp. 80-81). As such, the teacher's ability to engage in this shared and dynamic process is crucial. Matthews et al. (2013) build on this point noting that a task originally designed to be culturally relevant will quickly lose its relevance if the teacher does not implement it in a way that connects the task to students and their communities and supports students as they discuss and explore these connections. The sections below list essential sources to read and reflect on as you work to build your personal capacity to become a culturally responsive practitioner.

Explore and address your implicit bias

Research has shown that implicit bias, or the unconscious attitudes and stereotypes we hold, are prevalent in society, with educators being no exception. These implicit biases affect educational policies and are associated with negative academic outcomes for students of color through practices such as disproportionate use of exclusionary disciplinary and teachers' inequitable allocation of time and resources (Hu & Hancock, 2024).

To understand and work to address your implicit bias, explore these suggested readings:

- Recognizing Your Biases (National Education Association, 2024). https://www.nea.org/recognizing-your-biases
- Understanding Implicit Bias: What Educators Should Know (Staats, 2022). https://www.facinghistory.org/resource-library/understanding-implicit-bias-what-educators-should-know
- Racial Microaggressions in Everyday Life: Implications for Clinical Practice (Sue et al., 2007). https://www.cpedv.org/sites/main/files/file-attachments/how_to_be_an_effective_ally-lessons_learned_microaggressions.pdf

Learn about your students

In addition to addressing your implicit biases, culturally responsive teachers must "possess empowered, non-deficit views of their students" (Jones, 2015) and "learn about their students, their culture, and their backgrounds" (Johnson, 2009; Nieto, 2010).

To learn more about your students, their culture, and their backgrounds, explore these suggested readings:

- Funds of Knowledge Toolkit (Washington State Office of Superintendent of Public Instruction, 2023). https://ospi.k12.wa.us/sites/default/files/2023-10/funds_of_knowledge_toolkit.pdf
- Funds of Knowledge for Teaching: Using a Qualitative Approach to Connect Homes and Classrooms (Moll et al., 1992). https://education.ucsc.edu/ellisa/pdfs/Moll_Amanti_1992_Funds_of_Knowledge.pdf

Commit to praxis

Becoming a culturally responsive practitioner also requires teachers to engage in praxis, or the act of continually moving between theory, action, and reflection. To learn more about engaging in praxis as an educator, please see the suggested reading below:

- What is Praxis? (Stuart, 2020). https://sustainingcommunity.wordpress.com/2020/03/12/what-is-praxis/

Deepen your culturally responsive pedagogy knowledge base

To continue to deepen your theoretical knowledge base around culturally responsive instruction, please see the suggested readings below:

- Deconstructing Deficit Thinking: Working With Educators to Create More Equitable Learning Environments (García & Guerra, 2004). https://journals.sagepub.com/doi/epdf/10.1177/0013124503261322
- "But That's Just Good Teaching!" The Case for Culturally Relevant Pedagogy (Ladson-Billings, 1995). https://econedlink.org/wp-content/uploads/2020/06/LadsonBillings_Culturally-Relevant-Pedagogy.pdf
- Culturally Relevant Pedagogy 2.0: A.K.A. the Remix (Ladson- Billings, 2004). https://www.teachingworks.org/images/files/CRP_remix_HER.pdf
- Toward a Conceptual Framework of Culturally Relevant Pedagogy: An Overview of the Conceptual and Theoretical Literature (Brown-Jeffy & Cooper, 2011). https://files.eric.ed.gov/fulltext/EJ914924.pdf
- Culturally Sustaining Pedagogy: A Needed Change in Stance, Terminology, and Practice (Paris, 2012). https://journals.sagepub.com/doi/full/10.3102/0013189X12441244
- "Good Teachers" with "Good Intentions": Misappropriations of Culturally Responsive Pedagogy (Evans et al., 2020). https://files.eric.ed.gov/fulltext/EJ1283416.pdf

B. Engaged student and community funds of knowledge

Using engaged student and community funds of knowledge in middle school math instruction helps students bring their lived experience and intuitive knowledge to the instructional setting as assets for individual and collective learning (Aguirre & del Rosario Zavala, 2013; Jones, 2015; Turner et al., 2012). As part of this domain the teacher connects students' lives and experience outside of school to their work in the math classroom to advance their understanding of mathematical concepts. These connections should be built on the teacher's in-depth and evolving knowledge of students' lives and cultures and their authentic, supportive relationships with students.

Culturally responsive teaching domains

- **Engaged student and community funds of knowledge**
- Interdisciplinary connections
- Empowered mathematical inquiry and decision making

Practices and behaviors of the domain

To further understand what it looks like to use engaged student and community funds of knowledge in the classroom, review Exhibit 1. The table includes descriptions of the different practices and behaviors of this domain as well as counterexamples that do not illustrate the domain.

Exhibit 1. Key aspects of the engaged student and community funds of knowledge domain

Engaged student and community funds of knowledge		
Key aspect	**Example**	**Counterexample**
Assets for learning	Students' community funds of knowledge are treated as assets for learning.	Students' community funds of knowledge are treated as a distraction in the classroom.
	The teacher provides opportunities for students to share their knowledge about a topic and the connections they make between math class and their everyday lives.	*When students make connections between their lives and classwork, the teacher does not seek to learn more but instead corrects students for being distracted or off task.*
Trusting relationships	Teachers build trusting relationships with students to learn about their home and community lives to integrate this "knowledge base" into their teaching.	Teachers only know students at a superficial level and do not build relationships to learn about their home and community lives.
	The teacher spends time getting to know students' collective and individual cultures through a combination of structured activities and ad hoc connections with students.	*Teachers incorporate "getting to know you" activities at the beginning of the school year but do not develop relationships with students in other ways.*
Embedded math examples	Math examples are embedded in local community or cultural contexts.	The connections might be "culturally stereotypical," based on teachers' assumptions about students' lives outside of school, or "culturally neutral," based on widely known examples that are not directly applicable to students. Math examples include connections to real-world experiences but do not necessarily relate to students directly or are not connected to their lived experiences.
	The teacher includes a word problem that requires students to use an algebraic equation to compare home rental prices in their neighborhood.	*The teacher includes a word problem that uses students' names in place of generic names used by a curriculum developer.*
Mathematized student experiences	Students' experiences are mathematized, that is, used to advance their math understanding or skill. Student analyzes the mathematics at hand within their community, cultural, or linguistic context and in a way that advances the student's math understanding or skill.	Connections to students' experiences are made, but they do not advance their math understanding.
	Students are talking about the opening of a new Amazon warehouse near the school. The teacher incorporates this topic into lessons on rate of change by asking students to calculate how the new warehouse opening could affect local unemployment rates.	*The teacher discusses with students how they feel about the new Amazon warehouse but does not incorporate this topic into math lessons.*

Key questions

As you begin to learn about this domain, reflect on the following questions. These questions can catalyze your thinking about opportunities to adapt or design upcoming lessons or units to incorporate the domain into your instruction:

- How can I connect upcoming math content to my students' individual lived experience or local context? What generic information or data are required to perform the tasks associated with this lesson that could be replaced with information or data that are more relevant and relatable to my students?
- How could I model the use of this information or data?
- What questions could I ask my students to encourage them to use this information or data to answer my questions or solve a problem?
- What activities can I assign my students that require them to reason with their community or cultural knowledge?

What it looks like

To support using engaged student and community funds of knowledge in the classroom, you might think of how you can encourage the practices and student behaviors at three levels, which represent a progression of instruction from teacher- to student-led activities (see Exhibit 2). In the first level (**teacher led**), teachers demonstrate the practices and behaviors included in this domain. In the second level (**teacher facilitated**), teachers engage students in tasks that promote their use of the practices and behaviors. In the third level (**student led**), students independently use the practices and behaviors as they develop a shared understanding of mathematics and make sense of mathematical tasks.

Exhibit 2. Engaged student and community funds of knowledge domain at three levels

Engaged student and community funds of knowledge Draw on students' cultural and community funds of knowledge as learning assets		
Level 1: Teacher led Teachers demonstrate the practices and/or behaviors in this domain.	**Level 2: Teacher facilitated** Teachers engage students in tasks that promote their use of the practices and behaviors.	**Level 3: Student led** Students independently use the practices and behaviors as they develop a shared understanding of mathematics and make sense of mathematical tasks.
The teacher connects or employs students' community or cultural knowledge that is **specific to their individual lived experience or local context** with a math-related discussion or task.	The teacher incorporates discussions or tasks into their math lessons that ask students to connect or employ their cultural knowledge that is **specific to their individual lived experience or local context** with a math-related discussion or task.	The student independently connects or employs their community or cultural knowledge that is **specific to their individual lived experience or local context** with a math-related discussion or task.

Example lesson

On page 8 is an example of how a teacher adapted a lesson on balancing equations to incorporate engaged student and community funds of knowledge. The lesson is an excerpt of a full lesson from Illustrative Mathematics and has been annotated with examples of practices and behaviors at each of the three levels presented previously: teacher led, teacher facilitated, and student led. The example lesson is formatted in a specific way for ease of navigation:

Gray boxes provide a lesson overview and a summary of each of the sections of the lesson. These examples are taken directly from the developers' lesson and reflect their organization.

Red boxes show **Level 1, adaptations**
Teachers demonstrate the practices and behaviors of this domain.

Green boxes show **Level 2, adaptations**
Teachers engage students in tasks that promote use of the practices and behaviors.

Blue boxes show **Level 3, adaptations**
Students independently use the practices and behaviors as they develop a shared understanding of mathematics and make sense of mathematical tasks.

Lesson Overview

Illustrative Mathematics, Grade 8, Unit 4, Lesson 2

Lesson 2: Keeping the equation balanced

Learning goals:

- Calculate the weight of an unknown object using a hanger diagram and explain (orally) the solution method.
- Comprehend that adding and removing equal items from each side of a hanger diagram or multiplying and dividing items on each side of the hanger by the same amount are moves that keep the hanger balanced.

The original lesson includes the following sections:

- 2.1: Notice and Wonder: Hanging Socks
- 2.2: Hanging Blocks
- 2.3: More Hanging Blocks
- Lesson Synthesis
- 2.4: Cool down – *not included in the example below*

Section 2.1: Notice and Wonder: Hanging Socks

Warm-up

The purpose of this warm-up is to give students an opportunity to ground their understanding of equality in the context of weight, which is a context that will be used throughout the lesson.

Launch

Tell students they will look at a picture, and their job is to think of at least one thing they notice and at least one thing they wonder about the picture. Display the picture for all to see and give 1 minute of quiet think time.

Activity synthesis

Ask students to share their ideas. Record and display the responses for all to see. If not brought up, ask students why they think the left hanger is balanced while the right hanger is unbalanced. Students should understand that a hanger will only balance if the weight of the unknown objects in both socks is the same. If they are not the same, then the heavier side is lower than the lighter side.

In planning for this lesson, the teacher wants to replace the generic example of hangers with an example from students' lived experiences that draws on their everyday knowledge. There is a supermarket near the school that the teacher knows students and their families frequent. In addition, a number of students have older siblings that have recently gotten jobs packing groceries in the supermarket, something that the teacher and students discuss frequently. The teacher decides to use an example from the supermarket to illustrate equality in the context of weight. This example will be carried throughout the lesson.

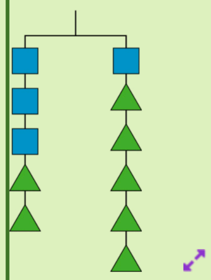
The teacher replaces the picture of hangers and socks with two pictures of people holding shopping bags—one bag in each hand. In one picture the bags are balanced and in the other the bags are unbalanced. The teacher then presents the scenario: two people are leaving the neighborhood supermarket with ingredients to cook chicken and rice for dinner. While each shopper bought the same items, their bags were packed by two different employees.

The teacher asks students what they notice about the two images, probing for why they think the bags of the person in one picture are balanced, while the bags of the other person are unbalanced. The teacher asks: What ingredients are needed to make chicken and rice? How much do you think these ingredients weigh, compared to each other? Which ingredients are heavy and which are light? Are there any ingredients that are large but also light or small but very heavy? How might these ingredients have been packed by the employee who helped the person whose bags are balanced? By the other employee?

Section 2.2: Hanging Blocks

Student Facing

This picture represents a hanger that is balanced because the weight on both sides is the same.



1. Elena takes two triangles off of the left side and three triangles off of the right side. Will the hanger still be in balance, or will it tip to one side? Which side? Explain how you know.

2. Use the applet to see if your answer to question [1] was correct. Can you find another way to make the hanger balance?

Activity

The purpose of this task is for students to understand and explain why they can add or subtract expressions from each side of an equation and still maintain the equality, even if the value of those expressions is not known. Both problems have shapes with unknown weight on each side to promote student thinking about unknown values in this way before the transition to equations.

While the focus of this activity is on the relationship between both sides of the hanger and not equations, some students may start the second problem by writing and solving an equation to find the weight of a square. While students are working, identify those using equations and those not using equations to answer the second problem during the whole class discussion.

Launch

Give 5 minutes of quiet work time followed by a whole-class discussion.

Activity synthesis

Begin the discussion by asking if students think the hanger will stay in balance, tip to the left, or tip to the right. Select 2–3 students to explain their vote. Make sure the class understands that removing unequal amounts of weight from the two sides results in the hanger tipping before moving on. Use Mathematical Language Routine 2 (Collect and Display) to capture student reasoning about it being okay to add or remove terms of the same "size" from both sides of an equation.

For the second question, select previously identified students to explain their answers, with the students who used equations going last. Record and display the specific equations the selected students wrote for all to see, such as $x + x + x + 1 + 1 = x + 1 + 1 + 1 + 1 + 1$ or $3x + 2 = x + 5$, and use it to help the class visualize how that student solved for the weight of a square.

The outcome of this discussion should be that it is okay to add or remove terms of the same "size" from both sides of an equation and the sides will still be equal. This can be thought of in terms of shapes hanging on hangers, where you can remove one square from both sides or add two triangles to both sides and the hanger will stay in balance. Equations are a more abstract representation of this, but the same concept holds: you can remove one x from both sides or add two 3s to both sides and the equation is still true with the left side equal to the right. Removing equal weights from both sides can leave the hanger with 2 squares on the left and 3 triangles (or just 3) on the right. In equation form, this is the same as $2x = 3$. Finally, you can halve the amount of weight on both sides of the hanger and keep it in balance, which is the same as multiplying $2x = 3$ by $1/2$ (or dividing both sides by 2).

The teacher tells students that they can think of the picture as a representation of the shopping bags in the warmup, with the shapes representing the different ingredients with different weights that they discussed. Throughout this section the teacher continues to frame the discussion in terms of the shopping bag example, and the shapes as representations of ingredients and their relative weights. Alternatively, the teacher can ask students to think of another example of objects with different weights, with the squares representing one of these objects, and the triangles representing the other. As the teacher facilitates the discussion they ask students to first share the example they thought of, and then explain if they think the hanger, or shopping bag, will stay in balance or not.

Section 2.3: More Hanging Blocks

Activity

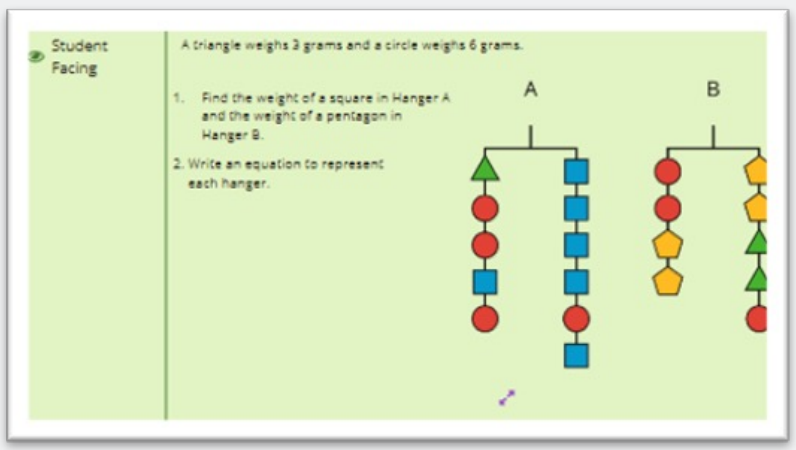
Building on the previous activity, students now solve two more hanger problems and write equations to represent each hanger. In the first problem, the solution is not an integer, which will challenge any student who has been using guess-and-check in the previous activities to look for a more efficient method. In the second problem, the solution is any weight, which is a preview of future lessons when students purposefully study equations with one solution, no solution, and infinite solutions. The goal of this activity is for students to transition their reasoning about solving hangers by maintaining the equality of each side to solving equations using the same logic. In future lessons, students will continue to develop this skill as equations grow more complex, culminating in solving systems of equations at the end of this unit.

As students work, identify those using strategies to find the weight of one square/pentagon that do not involve an equation. For example, some students may cross out pairs of shapes that are on each side (such as one circle and one square from each side of hanger A) to reason about a simpler problem while others may replace triangles with 3s and circles with 6s first before focusing on the value of 1 square. This type of reasoning should be encouraged and built upon using the language of equations.

Launch

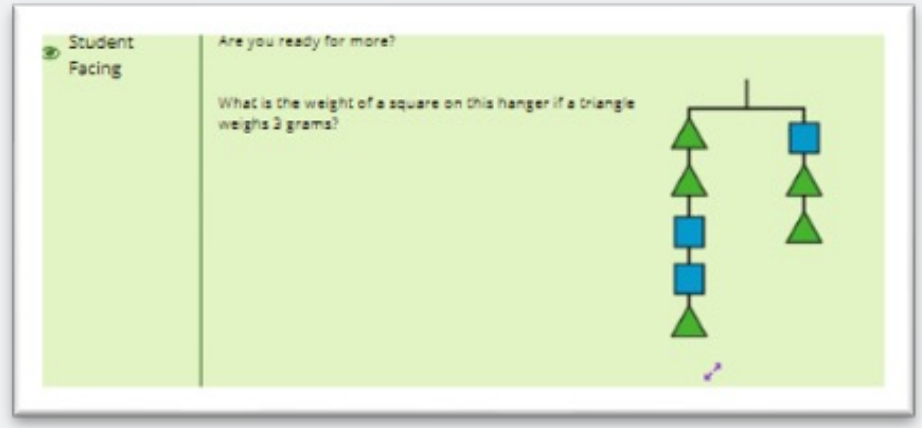
Arrange students in groups of 2. Give 5 minutes of quiet work time followed by partner discussion. Let students know that they should be prepared to share during the whole-class discussion, so they should make sure their partner understands and agrees with their solution.

Activity synthesis



Select previously identified students to share their strategies for finding the unknown weight without using an equation. Ask students to be clear how they are changing each side of the hanger equally as they share their solutions.

Next, record the equations written by students for each hanger and display for all to see in two lists. Assign half the class to the list for Hanger A and the other half to the list for Hanger B. Give students 1–2 minutes to examine the equations for their assigned hanger and be prepared to explain how different pairs of equations are related.



The goal here is for students to use the language they developed with the hangers (e.g., "remove 6 from each side") on equations.

For example, for Hanger A, you might contrast $3 + 6 + 6 + 6 = 4x + 6$ with $21 + x = 6 + 5x$. Possible student responses:

- Removing an x from each side of the second equation would result in the first equation.
- $x = 3.75$ makes both equations true.
- You can subtract 6s from the sides of each equation and they are still both true.

For Hanger B, examining equations should illuminate why it is impossible to know the weight of the unknown shape. If we start with $6 + 6 + x + x = x + x + 3 + 3 + 6$ and keep removing things of equal weight from each side, we might end up with an equation like $2x = 2x$. Any value of x will work to make this equation true. For example, if x is 10, then the equation is $20 = 20$. It is also possible to keep removing things of equal weight from each side and end up with an equation like $6 = 6$, which is always true.

The teacher tells students that this activity will build on the previous activity, and that they can continue to think of the pictures as representations of the shopping bags with ingredients for chicken and rice, or the other example of objects with different weights. Once students solve the problems and identify the values of the variables, the teacher asks them to connect this back to the example they used in Section 2.2. During the activity synthesis, the teacher asks students to share their example and strategies.

Lesson synthesis

The purpose of this discussion is to have students revisit the warm-up and connect it to the activities, reflecting on why the hanger is an appropriate and helpful analogy for an equation.

Ask these questions:

- "In the warm-up we wondered why one hanger was slanted, whether there were weights in one blue sock that made it heavier than the other, whether the crooked hanger would straighten out if another sock was added to the other side (add any other pertinent things your students wondered). How would you answer these questions now?"
- "What is an equation? What does the equal sign in an equation tell you?" (An equation is a statement that two expressions have the same value. The equal sign tells you that the expressions on either side must have the same value, however that value is measured—as a count of objects, a measurement like 10 miles or 6 seconds, or numbers without units).
- "What features do balanced hangers and equations have in common?" (Both representations have sides that are equal in value, even if the actual value of a side is unknown. Each side can contain numbers we do not know in the form of either shapes or variables. Changing the value of one side of a hanger or equations means changing the value of the other side by the same amount.)
- "You saw an example of a hanger where the unknown weight could not be determined. Can you design your own hanger like this one? How would you think about the weights needed on each side?" (If students completed the extension, you might ask them to also design a hanger with no solution.)

The teacher replaces the reflection questions with a performance tasks that asks students to independently create and solve a word problem on balancing equations that uses an example from their lives. The teacher offers support in scaffolding the exercise to help students who get stuck at various points.

Guiding questions for reflection and planning

After reviewing the example lesson, use the following questions to help you reflect on your current use of engaged student and community funds of knowledge as well as plan for how you can incorporate this domain into future lessons.

1. **Consider the example lesson.**
   a. Which adaptations resonated with you? Surprised you?
   b. What other ideas do you have for adapting the lesson to incorporate engaged student and community funds of knowledge?

2. **Consider your own instruction.**
   a. How do you currently incorporate engaged student and community funds of knowledge in your lessons?
   b. At what level of practice do you typically incorporate this domain (teacher led, teacher facilitated, or student led)? Why do you think this is?
   c. What are your strengths in terms of incorporating the practices in this domain?
   d. What are your areas for growth?
   e. What relationship building have you done, or do you need to do, in order to learn about students' lives, their cultural backgrounds, and issues in the community that you could incorporate into your instruction?

3. **Consider an upcoming lesson.**
   a. How might you incorporate examples from your students' lived experiences into your instruction (teacher led)?
   b. How do the lesson's tasks or questions encourage students to draw on their everyday knowledge and the activities they engage in at home or in their community? How might you adapt the tasks or questions to do this (teacher facilitated)?
   c. What classroom norms or structures can you incorporate to encourage students to independently connect their community or cultural funds of knowledge? What kinds of activities do you need to design for students so that they have this opportunity (student led)?

C. Interdisciplinary connections

Incorporating interdisciplinary connections in middle school math instruction requires the teacher to draw on connections from other content areas and domains of study to advance students' mathematical understanding of a fact, concept or procedure beyond the lesson (Aguirre & del Rosario Zavala, 2013; Jones, 2015; Turner et al., 2012). To use interdisciplinary connections the teacher should seek to learn about students' experiences in other classes and use this insight in their math classrooms to further students' mathematical understanding.

Culturally responsive teaching domains

- Engaged student and community funds of knowledge
- **Interdisciplinary connections**
- Empowered mathematical inquiry and decision making

Practices and behaviors of the domain

To further understand what it looks like to incorporate interdisciplinary connections in the classroom, review Exhibit 3. The table includes descriptions of the different practices and behaviors of this domain as well as counterexamples that do not illustrate the domain.

Key questions

As you begin to learn about this domain, reflect on the following questions. These can catalyze your thinking about opportunities to adapt or design upcoming lessons or units to incorporate the domain into your instruction:

- How can I connect upcoming math content to other academic disciplines or content areas such as history, science, or engineering that furthers students' understanding and improves their application of a mathematical fact, concept, or procedure beyond the lesson?
- How could I model using a connection to another discipline to broaden my mathematical understanding and application?
- What activities could I design that ask my students to use interdisciplinary connections to broaden their mathematical understanding and application?
- What activities could I assign that require students to use interdisciplinary connections independently?

What it looks like

To support using interdisciplinary connections in the classroom, think of the practices and student behaviors you can encourage at three instructional levels, from teacher- to student-led activities (see Exhibit 4).

Exhibit 3. Key aspects of the interdisciplinary connections domain with examples and non-examples

Interdisciplinary connections		
Key aspect	**Example**	**Counterexample**
Make connections to an academic discipline	Teachers make a connection to another academic discipline, such as biology, engineering, or architecture.	Teachers make a connection to a real-world example, as opposed to another academic discipline.
	The teacher connects a lesson on creating histograms to a project on climate change that students are working on in social studies. The teacher asks students to refer to an article from their social studies class on the concentration of greenhouse gasses over the past 20 years. The teacher asks students to create a histogram to visualize the data presented in the article.	*During a lesson on creating histograms, the teacher uses an example of a newspaper article showing how many calories are in typical Chipotle meals.*
Broaden students' understanding beyond the lesson	Teachers use interdisciplinary connections to broaden students' understanding beyond the lesson, supporting students to deepen their understanding of a mathematical fact, concept, or procedure.	Teachers use interdisciplinary connections, but these do not broaden students' understanding beyond the lesson.
	The teacher connects a lesson on exponential growth to students' social studies unit in which they are studying the demographics of different countries around the world. The teacher asks students to apply their knowledge of exponential growth to determine the population change of different countries. The teacher then asks students to think of what other information about different countries could be calculated using exponential growth.	*The teacher replaces the examples in a lesson on exponential growth with examples of population growth of different countries they are studying in social studies class, without connecting the examples back to the concept of exponential growth.*

Exhibit 4. Interdisciplinary connections domain at three levels

Interdisciplinary connections Make connections to other disciplines such as history, science, or engineering		
Level 1: Teacher led Teachers demonstrate the practices and/or behaviors in this domain.	**Level 2: Teacher facilitated** Teachers engage students in tasks that promote their use of the practices and behaviors.	**Level 3: Student led** Students independently use the practices and behaviors as they develop a shared understanding of mathematics and make sense of mathematical tasks.
The teacher connects a math-related discussion or task to **another academic discipline or content area** as a tool to broaden students' understanding and application of a mathematical fact, concept, or procedure beyond the lesson.	The teacher designs activities that ask students to connect a math-related discussion or task to **another academic discipline or content area** as a tool to broaden their understanding and application of a mathematical fact, concept, or procedure beyond the lesson.	The student independently connects a math-related discussion or task to **another academic discipline or content area** as a tool to broaden their understanding and application of a mathematical fact, concept, or procedure beyond the lesson.

Example lesson

On page 15 is an example of how a teacher adapted a lesson on calculating the area of composite figures to incorporate interdisciplinary connections. The lesson is an excerpt of a full lesson from Eureka Mathematics and has been annotated with examples of practices and behaviors at each of the three levels presented above: teacher led, teacher facilitated, and student led. The example also includes a rationale of how the teacher made decisions about the adaptations. The example lesson is formatted in a specific way for ease of navigation:

Gray boxes provide a lesson overview and a summary of each of the sections of the lesson. These examples are taken directly from the developers' lesson and reflect their organization.

Red boxes show **Level 1, adaptations**
Teachers demonstrate the practices and behaviors of this domain.

Green boxes show **Level 2, adaptations**
Teachers engage students in tasks that promote use of the practices and behaviors.

Blue boxes show **Level 3, adaptations**
Students independently use the practices and behaviors as they develop a shared understanding of mathematics and make sense of mathematical tasks.

Lesson overview

Eureka Mathematics, Grade 6, Topic A, Lesson 6

Area in the Real World

Student outcomes:

- Students determine the area of composite figures in real-life contextual situations using composition and decomposition of polygons.
- Students determine the area of a missing region using composition and decomposition of polygons.

Lesson Notes

Finding area in real-world contexts can be done around the classroom, in a hallway, or in different locations around the school. This lesson requires the teacher to measure and record the dimensions of several objects and calculate the area ahead of time. Choices are dependent on time available and various students' needs. Different levels of student autonomy can be taken into account when grouping and deciding which objects to measure. Further, the measurement units and precision can be adjusted to students' ability level.... Throughout the lesson, there are opportunities to compare unsimplified numerical expressions. These are important and should be emphasized because they help prepare students for algebra.

Classwork

Gauge students' ability level regarding which units and level of precision will be used in this lesson.... Choosing standard units allows precision to be set to the nearest foot, half foot, etc., but it could require multiplying fractional lengths.

Discussion (5 minutes)

- Area problems in the real world are all around us. Can you give an example of when you might need to know the area of something?
- The Problem Set from the last lesson had a wall that was to be painted. What measurement units were used in that problem?
- How precisely were the measurements made?
- Could those measurements have been made more precisely?
- We can measure the dimensions of objects and use those measurements to calculate the surface area of the object. Our first object will be a wall in this classroom.

In planning for this lesson, the teacher wants to replace the generic example of classroom wall paint with an example that makes a connection to another academic discipline. Through conversations with students and also discussions during Grade 6 team planning time, the teacher knows students are studying media literacy in social studies class and that they are very engaged during these lessons. The teacher decides to use an example of advertising content on social media feeds to calculate surface area. This example will be carried throughout the lesson.

The teacher revises the second part of the last discussion question and thinks out loud: "Thinking about calculating area in the real world got me thinking about discussions you are having in social studies class about advertising in print and social media. I wonder if we could use what we know about calculating the area of triangles, quadrilaterals, and polygons to determine how much of my social media feed is typically advertising content versus content posted by someone in my social network? This information could be helpful to me as I try and make informed decisions about the information I am seeing on social media."

Exploratory Challenge 1 (34 minutes): Classroom Wall Paint

Students make a prediction of how many square feet of painted surface there are on one wall in the room. If the floor has square tiles, these can be used as a guide.

Decide beforehand the information in the first three columns. Measure lengths and widths and calculate areas. Ask students to explain their predictions.

Exploratory Challenge 1 (34 minutes): Classroom Wall Paint

Exploratory Challenge 1: Classroom Wall Paint

The custodians are considering painting our classroom next summer. In order to know how much paint they must buy, the custodians need to know the total surface area of the walls. Why do you think they need to know this, and how can we find the information?

All classroom walls are different. Taking overall measurements and then subtracting windows, doors, or other areas will give a good approximation.

Make a prediction of how many square feet of painted surface there are on one wall in the room. If the floor has square tiles, these can be used as a guide.

Scaffolding:

This same context can be worded more simply for English language learners, and students working below grade level would benefit from a quick pantomime of painting a wall. A short video clip might also set the context quickly.

Estimate the dimensions and the area. Predict the area before you measure. My prediction: __________ ft^2.

a. Measure and sketch one classroom wall. Include measurements of windows, doors, or anything else that would not be painted.

Student responses will depend on the teacher's choice of wall.

Object or Item to Be Measured	Measurement Units	Precision (measure to the nearest)	Length	Width	Expression That Shows the Area	Area
Door	feet	half foot	$6\frac{1}{2}$ ft.	$3\frac{1}{2}$ ft.	$6\frac{1}{2}$ ft. $\times$ $3\frac{1}{2}$ ft.	$22\frac{3}{4}$ ft^2

b. Work with your partners and your sketch of the wall to determine the area that needs paint. Show your sketch and calculations below; clearly mark your measurements and area calculations.

c. A gallon of paint covers about $350\ \text{ft}^2$. Write an expression that shows the total area of the wall. Evaluate it to find how much paint is needed to paint the wall.

Answers will vary based on the size of the wall. Fractional answers are to be expected.

d. How many gallons of paint would need to be purchased to paint the wall?

Answers will vary based on the size of the wall. The answer from part (d) should be an exact quantity because gallons of paint are discrete units. Fractional answers from part (c) must be rounded up to the nearest whole gallon.

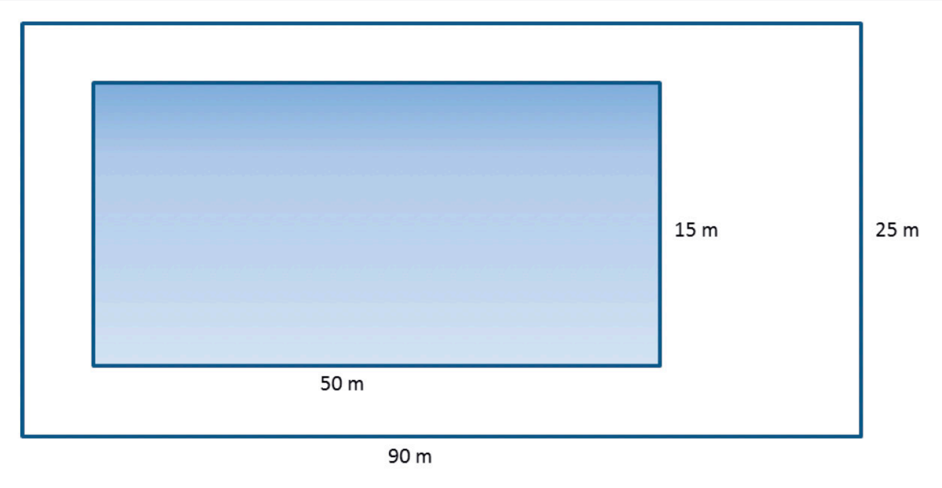
The teacher hands out a printout of a social media feed and asks students to circle everything they think is advertising content. The teacher has students share their responses and as a class they agree on what is advertising content and highlight or circle these areas. The teacher then asks students to predict how many square centimeters of the feed are advertising content.

The teacher adapts the rest of the exploratory challenge so that students are calculating the total area of advertising content on the printout by measuring the different shapes that contain advertising. Students then write equations that show how they would use these measurements to calculate the total area of advertising content on the feed.

Closing (3 minutes)

Exit Ticket

Find the area of the deck around this pool. The deck is the white area in the diagram.



The teacher replaces the exit ticket with a performance task that asks students to draw an object from another academic discipline they are studying and explain how they could calculate the area of this object.

Guiding questions for reflection and planning

After reviewing the example lesson, the following questions will help you reflect on your current use of interdisciplinary connections as well as plan for how you can incorporate this domain into future lessons.

1. **Consider the example lesson.**
   a. Which adaptations resonated with you? Surprised you?
   b. What other ideas do you have for adapting the lesson to incorporate interdisciplinary connections?
2. **Consider your own instruction.**
   a. How do you currently make connections to other academic disciplines or content areas in your lessons?
   b. At what level of practice do you typically incorporate this domain (teacher led, teacher facilitated, or student led)? Why do you think this is?
   c. What are your strengths in terms of incorporating the practices in this domain?
   d. What are your areas for growth?
3. **Consider an upcoming lesson.**
   a. How might you make connections to other academic disciplines or content areas into your instruction (teacher led)?
   b. How do the lesson's tasks or questions encourage students to make interdisciplinary connections to advance their mathematics understanding beyond the lesson? (teacher facilitated)?
   c. What performance tasks can you incorporate to encourage students to independently make interdisciplinary connections? What kind of support will students need to complete these performance tasks (student led)?

D. Empowered mathematical inquiry and decision making

Culturally responsive teaching domains

- Engaged student and community funds of knowledge
- Interdisciplinary connections
- Empowered mathematical inquiry and decision making

Incorporating empowered mathematical inquiry and decision making in middle school math instruction requires the teacher to support students as they use math to investigate or critique a social justice issue relevant to them. It involves the teacher choosing a social justice issue relevant to students (or letting students identify the issue); supporting students to use an empowerment lens in regard to the issue (one that recognizes their, and their communities agency within a situation); and using math as a tool to explore and critique the issue and the related economic, social, or political structures.

Practices and behaviors of the domain

To further understand of what it looks like to use empowered mathematical inquiry and decision making in the classroom, review Exhibit 5. The table includes descriptions of the different practices and behaviors of this domain as well as counterexamples that do not illustrate the domain.

Key questions

As you begin to learn about this domain, reflect on the following questions. These can catalyze your thinking about opportunities to adapt and/or design upcoming lessons or units to incorporate the domain into your instruction:

- How can I frame a lesson's tasks or questions around current or historical issues of social justice that may relate to the students in my classroom and help them use math as a tool to investigate or critique these issues?
- How could I model using math to investigate or critique these issues?
- What activities could I design that ask my students to use math as a tool to investigate or critique a social justice issue that they are concerned about?

Exhibit 5. Key aspects of the empowered mathematical inquiry and decision making domain with examples and non-examples

Empowered mathematical inquiry and decision making		
Key aspect	**Example**	**Counterexample**
Social justice issue of relevance	The societal issue is relevant to and/or chosen by students. Examples may include problems about population statistics, economic inequality, climate change, and climate justice.	The societal issue is not relevant to students and/or is chosen by the teacher with no student input.
	The teacher assigns an instructional task with a connection to a social justice issue such as population statistics, economic inequality, or climate justice that they heard students discussing.	*The teacher assigns an instructional task with a connection to a social issue that is outdated or not relevant to students' lives.*
Use of an empowerment lens	Math tasks feature an empowerment lens (versus deficit or color-blind orientation) applied to students' culture.	Math tasks use a deficit or color-blind lens, making assumptions and trading on negative stereotypes about students' culture.
	The teacher assigns an instructional task that highlights Ojjibwe, Oneida, and Menominee legends while encouraging students to build an understanding of one-step equations.	*The teacher assigns an instructional task that may highlight a culture unrelated to any students in the classroom because they did not make connections with the students. Or the teacher intentionally avoids highlighting any culture in an attempt to keep all equal.*
Math as a tool to investigate or critique social justice issues	Students are asked to use math as an analytical tool to inquire about "themselves, their communities, and the world around them." Students explore and existing structures (such as social or economic) and use math to critique and change these structures.	Social justice issues are incorporated into the lesson but are incorporated as content modifications instead of as a pathway to use math to investigate and/or critique the issues.
	The teacher assigns an instructional task that encourages students to review, list, and visualize department of corrections data in order to suggest alternative policies to reduce the racial disparities shown in the data.	*The teacher assigns an instructional task that illustrates racial disparities in department of corrections data but does not ask students to investigate or critique this disparity.*

What it looks like

To support using empowered mathematical inquiry and decision making in the classroom, think of the practices and student behaviors you can encourage at three instructional levels, from teacher- to student-led activities (see Exhibit 6).

Exhibit 6. Empowered mathematical inquiry and decision making domain at three levels

Empowered mathematical inquiry and decision making Explore social justice issues relevant to students using math as a tool		
Level 1: Teacher led Teachers demonstrate the practices and/or behaviors in this domain.	**Level 2: Teacher facilitated** Teachers engage students in tasks that promote their use of the practices and behaviors.	**Level 3: Student led** Students independently use the practices and behaviors as they develop a shared understanding of mathematics and make sense of mathematical tasks.
The teacher connects a math-related discussion or task to **another academic discipline or content area** as a tool to broaden students' understanding and application of a mathematical fact, concept, or procedure beyond the lesson.	The teacher poses a question, initiates a discussion, or assigns an instructional task that requires students to use math to **investigate or critique a societal challenge or a social justice issue** directly relevant to them or of their own choosing.	The student independently use math to **investigate or critique a societal challenge or a social justice issue** directly relevant to them or of their own choosing.

Example lesson

On page 20 is an example of how a teacher adapted a lesson on estimating probabilities by collecting data to incorporate empowered mathematical inquiry and decision making. The lesson is an excerpt of a full lesson from Eureka Mathematics and has been annotated with examples of practices and behaviors at each of the three levels presented above: teacher-led, teacher-facilitated, and student-led activities. The example also includes a rationale of how the teacher made decisions about the adaptations. The example lesson is formatted in a specific way for ease of navigation:

Gray boxes provide a lesson overview and a summary of each of the sections of the lesson. These examples are taken directly from the developers' lesson and reflect their organization.

Red boxes show **Level 1, teacher-led adaptations**
Teachers demonstrate the practices and/or behaviors this domain.

Green boxes show **Level 2, teacher-facilitated adaptations**
Teachers engage students in tasks that promote use of the practices and behaviors.

Blue boxes show **Level 3, student-led adaptations**
Students independently use the practices and behaviors as they develop a shared understanding of mathematics and make sense of mathematical tasks.

Lesson overview

Eureka Mathematics, Grade 7, Module 5, Lesson 2

Estimating probabilities by collecting data

Student outcomes

- Students estimate probabilities by collecting data on an outcome of a chance experiment.
- Students use given data to estimate probabilities.

Lesson overview:

This lesson builds on students' beginning understanding of probability. Lesson 1 introduced students to an informal idea of probability and the vocabulary *impossible*, *unlikely*, *equally likely*, *likely*, and *certain* to describe the chance of an event occurring. In this lesson, students begin by playing a game similar to the game they played in Lesson 1. The results of the game are used to introduce a method for finding an estimate for the probability of an event occurring. Then, students use data given in a table to estimate the probability of an event.

The lesson has the following sections

- Exercises 1–6 (18 minutes): Carnival game
- Exercises 7–8 (5 minutes)
- Example 2 (10 minutes): Animal crackers
- Exercises 9–15 (5 minutes)
- Closing (2 minutes)
- Exit ticket

Exercises 1–6: Carnival game

Place students into groups of two. Hand out a copy of the spinner and a paper clip to each group. Read through the rules of the game and demonstrate how to use the paper clip as a spinner.

Before playing the game, display the probability scale from Lesson 1 and ask students where they would place the probability of winning the game.

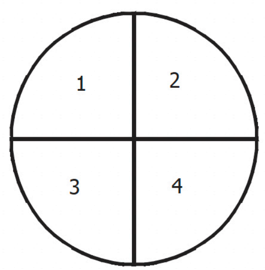
Remind students to carefully record the results of each spin.

Students work with their partners on Exercises 1–8. Then discuss and confirm as a class.

Sample responses to the questions should be based on the outcomes recorded by students. The following outcomes were generated by two students. They are used to provide sample responses to the questions that follow.

Exercises 1–8: Carnival Game

At the school carnival, there is a game in which students spin a large spinner. The spinner has four equal sections numbered 1–4 as shown below. To play the game, a student spins the spinner twice and adds the two numbers that the spinner lands on. If the sum is greater than or equal to 5, the student wins a prize.



The teacher replaces the example of a spinner with an example of data on racial discipline disparities from a nearby school district.

Play this game with your partner 15 times. Record the outcome of each spin in the table below.

Turn	1st Spin Results	2nd Spin Results	Sum
1	4	1	5
2	1	3	4
3	3	2	5
4	1	1	2
5	2	1	3
6	1	4	5
7	4	1	5
8	3	1	4
9	2	4	6
10	4	4	8
11	1	1	2
12	4	3	7
13	3	4	7
14	3	1	4
15	1	2	3

1. **Out of the 15 turns, how many times was the sum greater than or equal to 5?**

   Answers will vary and should reflect the results from students playing the game 15 times. In the example above, eight outcomes had a sum greater than or equal to 5.

2. **What sum occurred most often?**

   5 occurred the most.

3. **What sum occurred least often?**

   6 and 8 occurred the least. (Anticipate a range of answers, as this was only done 15 times. We anticipate that 2 and 8 will not occur as often.)

4. **If students were to play a lot of games, what fraction of the games would they win? Explain your answer.**

   Based on the above outcomes, $\frac{8}{15}$ *represents the fraction of outcomes with a sum of 5 or more. To determine this, count how many games have a sum of 5 or more. There are 8 games out of the total 15 that have a sum of 5 or more.*

5. **Name a sum that would be impossible to get while playing the game.**

   Answers will vary. One possibility is getting a sum of 100. Any sum less than 2 or greater than 8 would be correct.

6. **What event is certain to occur while playing the game?**

   Answers will vary. One possibility is getting a sum between 2 and 8 because all possible sums are between 2 and 8, inclusive.

Exercises 7–8

Before students work on Exercises 7 and 8, discuss the definition of a chance experiment. A chance experiment is the process of making an observation when the outcome is not certain (that is, when there is more than one possible outcome). If students struggle with this idea, present some examples of a chance experiment such as flipping a coin 15 times or selecting a cube from a bag of 20 cubes. Then display the formula for finding an estimate for the probability of an event. Using the game that students just played, explain that the denominator is the total number of times they played the game, and the numerator is the number of times they recorded a sum greater than or equal to 5.

When you were spinning the spinner and recording the outcomes, you were performing a *chance experiment*. You can use the results from a chance experiment to estimate the probability of an event. In Exercise 1, you spun the spinner 15 times and counted how many times the sum was greater than or equal to 5. An estimate for the probability of a sum greater than or equal to 5 is

$$P(\text{sum } \geq 5) = \frac{\textbf{Number of observed occurrences of the event}}{\textbf{Total number of observations}}.$$

Give students a few minutes to answer Exercises 7 and 8 and then ask each group to share their results. After students have shared their results, point out that not every group had exactly the same answer.

Ask students to explain why their answers are estimates of the probability of getting a sum of 5 or more.

Students will learn how to determine a theoretical probability for problems similar to this game. Before they begin determining the theoretical probability, however, summarize how an estimated probability is based on the proportion of the number of specific outcomes to the total number of outcomes. Students may also begin to realize that the more outcomes they determine, the more confident they are that the proportion of winning the game is providing an accurate estimate of the probability. These ideas are developed more fully in the following lessons.

7. **Based on your experiment of playing the game, what is your estimate for the probability of getting a sum of 5 or more?**

   Answers will vary. Students should answer this question based on their results. For the results indicated above, $\frac{8}{15}$ ***or approximately*** 0.53 ***or*** 53% ***would estimate the probability of getting a sum of*** 5 ***or more.***

8. **Based on your experiment of playing the game, what is your estimate for the probability of getting a sum of exactly 5?**

   Answers will vary. Students should answer this question based on their results. Using the above 15 ***outcomes,*** $\frac{4}{15}$ ***or approximately*** 0.27 ***or*** 27% ***of the time represents an estimate for the probability of getting a sum of exactly*** 5.

Example 2: Animal crackers

Students read the example. Consider showing a box of animal crackers and demonstrating how a student can take a sample from the box. Explain that the data presenting resulted from a student taking a sample of 20 crackers from a very large jar of animal crackers and recording the results for each draw.

Display the table of data.

Ask students:

- What was the total number of observations?
- How many zebras were chosen?
- What is the estimate for the probability of selecting a zebra?

The main point of this example is for students to estimate the probability of selecting a certain type of animal cracker. Use the data collected to make this estimate.

Example 2: Animal Crackers

A student brought a very large jar of animal crackers to share with students in class. Rather than count and sort all the different types of crackers, the student randomly chose 20 crackers and found the following counts for the different types of animal crackers. Estimate the probability of selecting a zebra.

Animal	Number Selected
Lion	2
Camel	1
Monkey	4
Elephant	5
Zebra	3
Penguin	3
Tortoise	2
	Total 20

The estimated probability of picking a zebra is $\frac{3}{20}$, or 0.15 or 15%. This means that an estimate of the proportion of the time a zebra will be selected is 0.15 or 15% of the time. This could be written as $P(\text{zebra}) = 0.15$, or the probability of selecting a zebra is 0.15.

The teacher replaces the animal cracker example with student disciplinary data from a local school district. The teacher explains that they calculated the number of middle school students who were suspended from school last year and broke this data down by the students' race. Explain that they then represented this data using different colored game pieces in a box. Explain that the data presented in the table resulted from a student taking a sample of 100 game pieces from the box and recording the results for each draw.

Student race	Number selected (suspended)
Hispanic (blue game piece)	37
Black (green game piece)	52
White (red game piece)	6
American Indian, multi-racial, and unidentified (purple game piece)	5
Total	100

Exercises 9–15

Place students in groups of two to answer each question. Consider specifying in which form they should answer. For this exercise, it is acceptable for students to write answers in fraction form to emphasize the formula. As a class, briefly discuss students' answers. Specifically, discuss the answer for Exercise 11. This question involves "or." For this question, students should indicate that they would add the outcomes as indicated in the question to form their proportions.

Exercises 9–15

If a student randomly selected a cracker from a large jar:

9. **What is your estimate for the probability of selecting a lion?**

$$\frac{2}{20} = \frac{1}{10} = 0.1$$

10. **What is your estimate for the probability of selecting a monkey?**

$$\frac{4}{20} = \frac{1}{5} = 0.2$$

11. **What is your estimate for the probability of selecting a penguin or a camel?**

$$\frac{(3+1)}{20} = \frac{4}{20} = \frac{1}{5} = 0.2$$

12. **What is your estimate for the probability of selecting a rabbit?**

$$\frac{0}{20} = 0$$

13. **Is there the same number of each kind of animal cracker in the jar? Explain your answer.**

No. There appears to be more elephants than other types of crackers.

14. **If the student randomly selected another 20 animal crackers, would the same results occur? Why or why not?**

Probably not. Results may be similar, but it is very unlikely they would be exactly the same.

15. **If there are 500 animal crackers in the jar, approximately how many elephants are in the jar? Explain your answer.**

$\frac{5}{20} = \frac{1}{4} = 0.25$; *hence, an estimate for the number of elephants would be* 125 *because* 25% *of* 500 *is* 125.

Closing and exit ticket

What information do you need to determine the estimated probability?

Discuss the Lesson Summary with students. Ask students to summarize how they would find the probability of an event.

Lesson Summary

An estimate for finding the probability of an event occurring is

$$P(\text{event occurring}) = \frac{\text{Number of observed occurrences of the event}}{\text{Total number of observations}}$$

The teacher asks the students to generate an example of data that show a disparity by race, gender, socioeconomic status, or another category that they have experienced or noticed in the world around them. The students are asked to create a table with these data, summarize how they would find the probability of an event, and then calculate the probability of this event.

Guiding questions for reflection and planning

After reviewing the example lesson, the following questions will help you reflect on your current use of empowered mathematical inquiry and decision making as well as plan for how you can incorporate this domain into future lessons.

1. **Consider the example lesson.**
   a. Which adaptations that the teacher made resonated with you? Surprised you?
   b. What other ideas do you have for adapting the lesson to incorporate empowered mathematical inquiry and decision making?
2. **Consider your own instruction.**
   a. Do you currently incorporate empowered mathematical inquiry and decision making in your lessons? If so, what are things you typically do or consider?
   b. If not, what are some ideas you have that you might incorporate in the future?
   c. At what level of practice do you typically incorporate this domain (teacher led, teacher facilitated, or student led)? Why do you think this is?
   d. What relationship building have you done, or do you need to do, in order to learn about issues of social justice that are relevant to students? What work have you done to address your implicit bias so that you can take an empowered view of this issue?
3. **Consider an upcoming lesson.**
   a. How might you incorporate empowered mathematical inquiry and decision making into your instruction (teacher led)?
   b. How do the lesson's tasks or questions encourage students to examine and critique a social justice issue relevant to them? How might you adapt the tasks or questions to do this (teacher facilitated)?
   c. What classroom norms or structures can you incorporate to encourage students to independently explore and critique social justice issues? What kinds of activities do you need to design for students so that they have this opportunity (student led)?

E. Potential misappropriations

While use of culturally responsive teaching has been found to improve the classroom experiences of students from historically marginalized backgrounds, researchers have also found that misuse of these practices can have harmful effects on the students they are meant to support. According to Evans et al., educators can misappropriate culturally responsive teaching when they use this pedagogy "to satisfy policy, funding, and reform mandates" as opposed to using it as "a means of establishing critically conscious and highly effective education" (Evans et al., p. 54, 2020). This misappropriation results in educators' perpetuation of harmful stereotypes and ineffective practices that can ultimately harm the educational experiences and outcomes of students of color (Evans et al., 2020). To build educators' understanding of how culturally responsive teaching can be misappropriated, we have included examples of misappropriation for each of the example lessons. Our examples use three common types of misappropriation defined by Evans et al.: 1) Those that are used as a "smokescreen of good intentions" by well-meaning teachers who can then avoid "deeply interrogating what they believe about teaching students who do not share the same cultural knowledge, social expectations, and language practices;" 2) Those that use culture as a hook solely to gain students' attention without using it as an "avenue" to further students' learning; and 3) Those that use culturally responsive teaching as a tool of assimilation by viewing students' lives and cultural funds of knowledge as deficits to be corrected.

Engaged student and community funds of knowledge example lesson
Illustrative Mathematics, Grade 8, Unit 4, Lesson 2: Keeping the equation balanced

Tool of assimilation. In this lesson the teacher asks students to share examples of balance from their personal lives and explore how they can then write equations to represent this balance. If, when they shared their examples, the teacher proceeded to correct students or negate the examples they were sharing this potentially culturally responsive lesson could become a tool of assimilation in which, instead of integrating this experience into the classroom context, they used it as an opportunity to pass judgement on the students' home culture.

Interdisciplinary connections example lesson
Eureka Mathematics, Grade 6, Topic A, Lesson 6: Area in the Real World

Culture as a hook. In this lesson, the teacher connects the mathematical concepts to social media, a topic the teacher knows they are studying in another class, and that is also prevalent in their everyday lives. If the teacher had simply introduced an example of advertisements in social media, as opposed to asking students to use math to explore this throughout the lesson, they would have missed a valuable opportunity to use a prominent part of students' culture to advance their mathematical understanding.

Empowered mathematical inquiry and decision making example lesson
Eureka Mathematics, Grade 7, Module 5, Lesson 2: Estimating probabilities by collecting data

Smokescreen of good intentions. In this lesson the teacher asks students to explore disproportionality in school discipline. The teacher has decided to incorporate this issue into their instruction because they believe it is of interest to students and they will have strong opinions about it. However, if this teacher has not adequately explored their implicit biases and the systemic nature of this issue, they may believe that students of color are more often the recipients of exclusionary discipline than their white or Asian peers because they are engaging in more "bad" behavior, as opposed to the unequal and subjective application of disciplinary policies that research has found negatively impact Black and Latinx students in particular. Attempting to incorporate this issue into a lesson while holding this belief, while "well intentioned," could perpetuate negative stereotypes.

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Appendix A: Background of the AMS study and rationale for this guide

The AMS study hypothesizes that culturally responsive mathematics teaching supports students who are Black, Latino, English learners, or experiencing poverty—and students who combine these identities and contexts—to have a better classroom experience. The approach aims to improve students' mathematics enjoyment, achievement identity, performance, persistence, self-efficacy, and growth mindset. Other studies support the AMS view. A study with more than 25,000 Tucson students found that participation in a Mexican American Studies class, which included practices aligned with those in the culturally responsive domains, positively affected by 8.7 percent the probability of passing the Arizona state math test among students who had previously failed it (Cabrera et al., 2014). Another study in 2017 found that participation in an ethnic studies course among grade 9 students whose prior year grade point average (GPA) was below 2.0 increased their grade 9 GPA, attendance, and credits earned at a greater rate than similar students who did not take the ethnic studies course (Dee & Penner, 2017). Use of practices that research has shown can improve the mathematics achievement of students from historically marginalized groups is of continued importance; a recent report by the National Assessment of Educational Progress found that, while mathematics scores among 13-year-olds declined for most student groups in the period from 2020-2023, the racial achievement gap between Black and White students widened from 35 points in 2020 to 42 points in 2023 (National Assessment of Educational Progress, 2023).

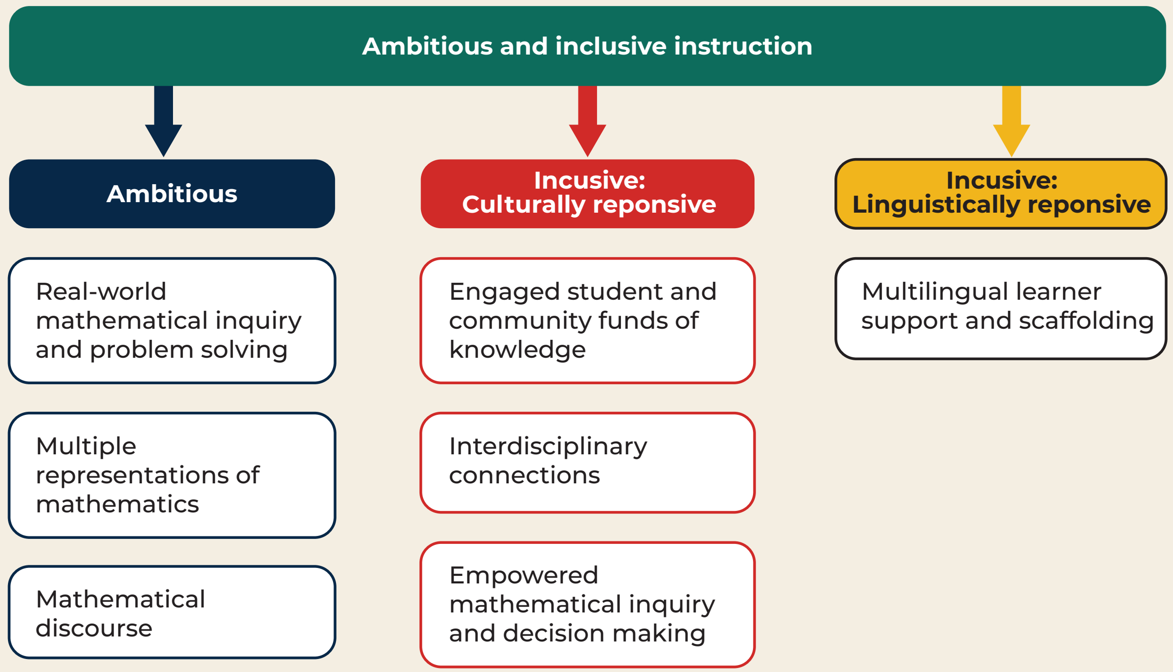
The AMS Study is part of a larger set of investments by the Bill & Melinda Gates Foundation intended to help students who are Black, Latino, multilingual learners, and/or experiencing poverty succeed in mathematics. This study's contribution to that work is to investigate the enabling and disabling conditions under which teachers incorporate culturally responsive teaching practices into their mathematics instruction. The AMS study aimed to understand the extent to which teachers in four urban school districts planned and executed ambitious and inclusive mathematics lessons while using one of six different middle school mathematics curricula.

The study used a variety of data collection methods, including observation of classroom practice, systematic coding of curricula, and surveys and interviews of teachers. Although the study team's classroom observations revealed that teachers use of culturally responsive practices was limited, most teachers reported receiving professional learning in this topic area and frequently adapting their instruction to incorporate these practices.

Seven domains of ambitious and inclusive mathematics instruction

Ambitious and inclusive mathematics instruction includes the content knowledge, dispositions, and practices that collectively promote mathematical thinking, cultural and linguistic funds of knowledge, and social justice (Aguirre & del Rosario Zavala, 2013). The AMS study organizes ambitious and inclusive instruction into seven domains across three categories: (1) ambitious instruction that promotes real-world mathematical inquiry and problem solving, use of multiple representations of mathematics, and mathematical discourse; (2) culturally responsive instruction in which the teacher promotes engaged student and community funds of knowledge, interdisciplinary connections, and empowered mathematical inquiry and decision making; and (3) linguistically responsive instruction that includes language support and scaffolding for multilingual learners (see Exhibit 1 below).

Exhibit 1



This practice guide aims to bridge the gap between professional learning and implementing culturally responsive instruction, specifically for the culturally responsive practices that were observed least frequently, by providing tools to help teachers plan and execute culturally responsive mathematics lessons in middle school classrooms.

AMS study classroom observations found limited use of culturally responsive practices

- Over the course of two consecutive school years, the AMS study team collected data from more than 100 teachers across the four school districts and conducted nearly 90 observations of classroom practice from a subset of those teachers to understand the extent to which teachers are instructing in ambitious and inclusive ways.
- Across the observed lessons, there were varying degrees of incorporation of each of the seven domains: the team observed teachers incorporated ambitious domains more frequently than the culturally and linguistically inclusive domains (see chart below). In addition, AMS study analysis of six curricula revealed that they contained limited explicit guidance for enacting culturally responsive practices.