Mathematica
Progress Together

Math curriculum adaptations

A study of productive alignment with mathematical learning progressions

Riley Stone, Doug Van Dine, and Jose Aguayo

BILL & MELINDA GATES foundation

https://shorturl.at/DaRQK

Broader AMS project overview

Analysis of Middle School Math Systems

- 6 standards-based curricula: 3 “high quality” and 3 “business-as-usual”
- 4 urban school districts, approximately 150 schools, 12 “deep dive” schools (36 classrooms)
- Students of color, multilingual learners, and students experiencing poverty

Enactment study

- Mixed methods design to explore the influence of curriculum design, professional learning, district and school instructional contexts, and teacher characteristics on:
  - **Curricular adherence and adaptation**
  - Ambitious and inclusive instructional practice
  - Student math identity, persistence, enjoyment, self-efficacy, engagement, growth mindset and performance

Broader project findings

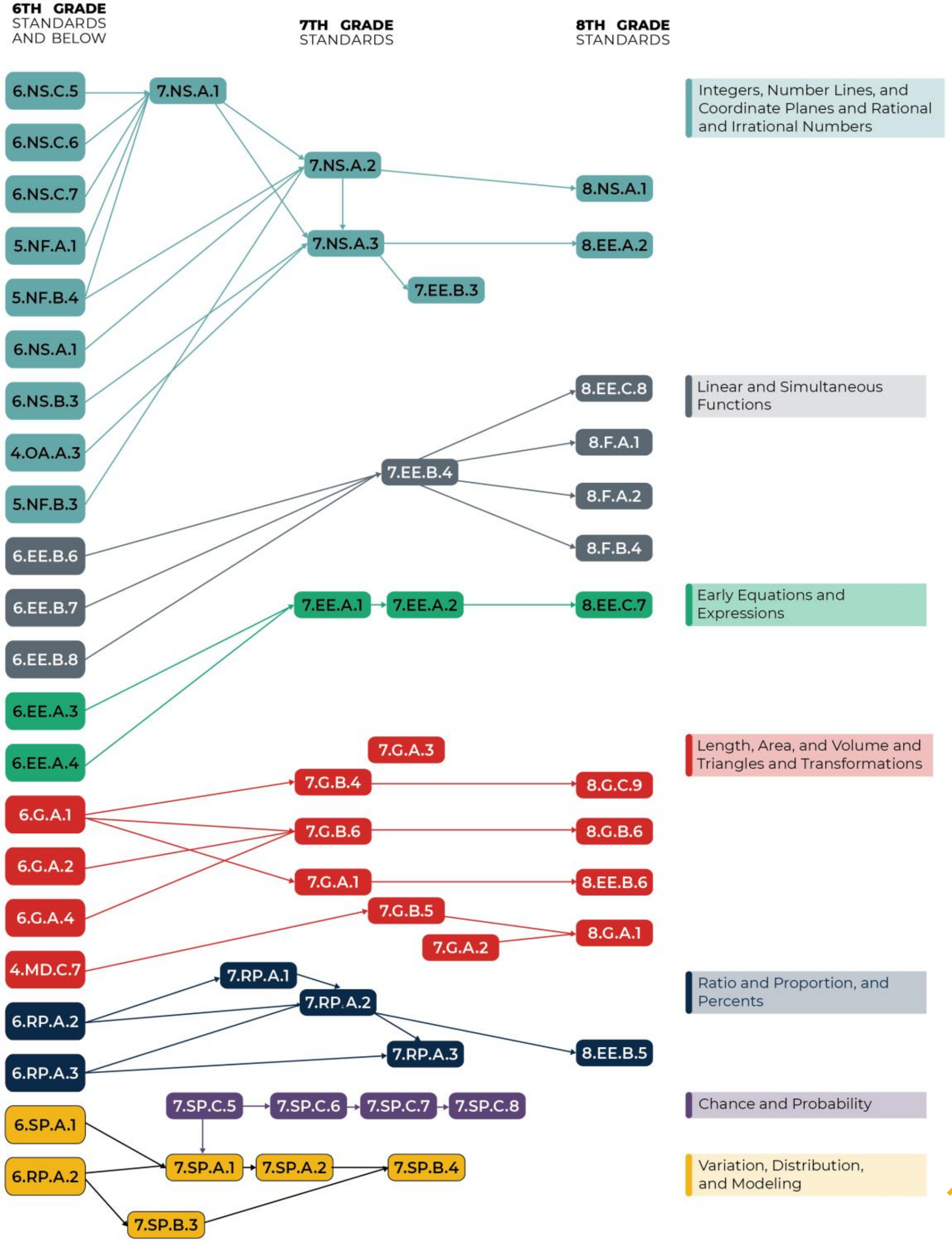
- We found that, regarding curricula…
  - Study curricula have **less cognitively demanding tasks** than the CCSS recommends.
  - Study curricula focus more on Operations, Data Displays, and Measurement topics, **than Algebra, Probability, and Statistics topics** in comparison to the CCSS.
- We found that, regarding adaptations…
  - **Most teachers report adapting lessons at least a few times a week.**
    - Adapters usually change the way content is delivered, the sequence, or by removing content or materials.
    - **These could potentially compromise rigor.**
  - A majority of adapters modify lessons to ensure a more equitable experience for their students. For example:
    - Differentiating instruction for students **performing below grade level and multilingual learners.**
    - Differentiating instruction for students **performing above grade level.**

What motivated the study?

- After reflecting on the first year of data, we found many teachers deviated from the or supplemented their core curriculum and wondered whether those shifts were hurting or helping students.
- We wanted to explore whether teachers made adaptations that aligned with or moved students away from intended learning progressions.

What are mathematical learning progressions?

Clusters of knowledge that identify pathways students follow as they develop more sophisticated ways of reasoning about content in mathematical domains within and spanning across grade bands.



Sources: Coherence Map; A Graph of the Content Standards

Why are mathematical learning progressions important?

- There is **limited evidence** for whether teachers consider mathematical learning progressions
- Teacher's adaptations have an **impact on students' experiences and achievement**
- Mathematical learning progressions are based on **research on children's cognitive development** and the structure of mathematics

Thus, adaptations in alignment with mathematical learning progression influence students' experiences and achievement.

Our research questions and methodology

RQ1

Do teachers make productive adaptations to their lessons; that is, do adaptions follow mathematical learning progressions?

RQ2

How often are teachers' adaptations below, on, or above grade level?

RQ3

How do teachers describe their adaptations and why they make these adaptations?

Methodology

- RQ1: Descriptive analyses of all lessons
- RQ2: Descriptive analyses of lessons with adaptations
- RQ3: Thematic analysis of teacher interviews

Identifying content adaptations

Intended standards
- Identified in the materials districts purchase from product developers

Planned standards
- Planned as the focus when a teacher prepares the lesson

Delivered standards
- Observed while the teacher delivers the lesson

8

Example of productive, ***below-grade-level*** adaptations

Productive, **below grade-level adaptations** generally resulted in lessons that focused on a prerequisite concept that should be mastered before the intended topic.

Productive, below grade-level

Example

In 7th grade *Illustrative Math*, an intended standard is to apply and extend previous understandings of addition and subtraction to add and subtract rational numbers **(7.NS.A.1)**[a]. The teacher adapted the lesson by focusing on ordering rational numbers and understanding absolute values **(6.NS.C.7)**[a].

Why is it productive?

These standards both fall within the *Rational and Irrational Numbers* learning progression.[b]

Example of productive, ***below-grade-level*** adaptations

Productive, below grade-level

Example

In 8th grade *Eureka Math*, an intended standard is to graph proportional relationships interpreting the unit rate as the slope of a graph **(8.EE.B.5)**[a]. The teacher adapted the lesson, focusing on understanding proportional relationships between quantities **(7.RP.A.2)**[a] which is prerequisite to interpreting the unit rate as the slope.

Why is it productive?

These standards both fall within the *Rational and Irrational Numbers* learning progression.[b]

Example of nonproductive, ***below-grade-level*** adaptations

Exhibit 13. Nonproductive, below grade-level

Nonproductive, below grade-level adaptations resulted in lessons that covered elementary content that did not align with middle school content.

Nonproductive, below grade-level

Example

In 6th grade Into Math, an intended standard is to draw polygons, find side lengths, and solve real-world math problems **(6.G.A.3)[a].** While enacting the lesson, the teacher had students find logos (such as car or shoe logos) and describe the characteristics of the geometric shapes in the logo which aligns with understanding shapes and their attributes **(3.G.A.1)**[a].

Why is it nonproductive?

6.G.A.3 falls on the *Length, Area, and Volume* learning progression.[b]

3.G.A.1 falls on the *Shapes and Angles* learning progression.[b]

Example of productive, ***on-grade-level*** adaptations

Productive, on-grade-level adaptations generally emphasized grade-level content that was foundational to meeting the intended standard.

Productive, on grade-level

Example

A teacher adapted the 8th grade KEMS standard that has students analyze and solve pairs of simultaneous linear equations **(8.EE.C.8.)**[a] by focusing a majority of the lesson on graphing a line using the slope and y-intercept **(8.EE.B.6)**[a].

Why is it productive?

These standards both fall within the *Rational and Irrational Numbers* learning progression.[b]

Example of nonproductive, ***on-grade-level*** adaptations

Nonproductive, **on-grade-level adaptations** were generally instances where teachers deviated from the curriculum and the content in the lesson did not cover the intended standard.

Nonproductive, on grade-level

Example

In 8th grade *Eureka Math*, an intended standard is to estimate where square roots fall on a number line and the meaning of square roots **(8.NS.A.1** & **8.NS.A.2)**[a]. However, the teacher asked students to solve equations with square roots using a calculator **(8.EE.A.2)**[a].

Why is it nonproductive?

8.NS.A.1 & **8.NS.A.2** fall on the *Rational and Irrational Numbers* learning progression.[b]

8.EE.A.2 falls on the *Linear and Simultaneous Functions* learning progression.[b]

We found that…

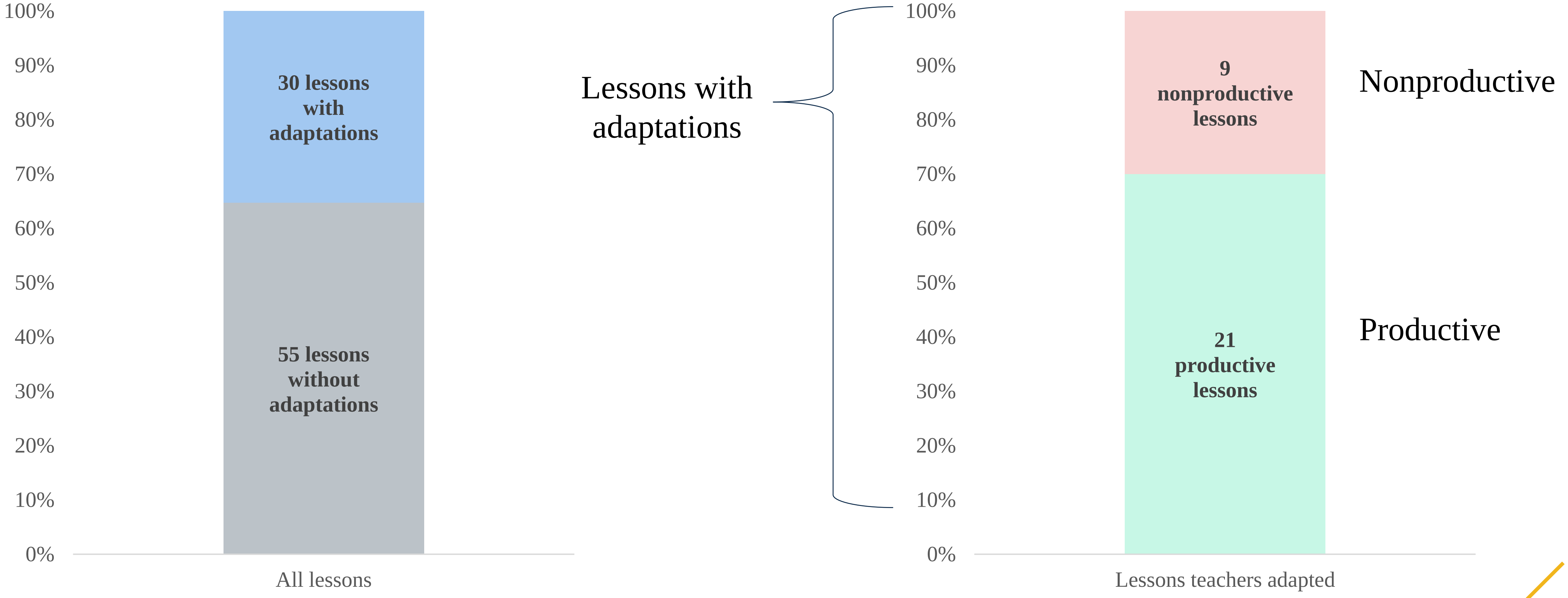
1. Teachers **made productive adaptations more often** than nonproductive ones.

2. Teachers made productive adaptations more often while **planning a lesson** and nonproductive adaptations more often while **delivering a lesson**.

3. Teachers made productive adaptations more often to content **below grade level** and made nonproductive adaptions more often to content **on grade level**.

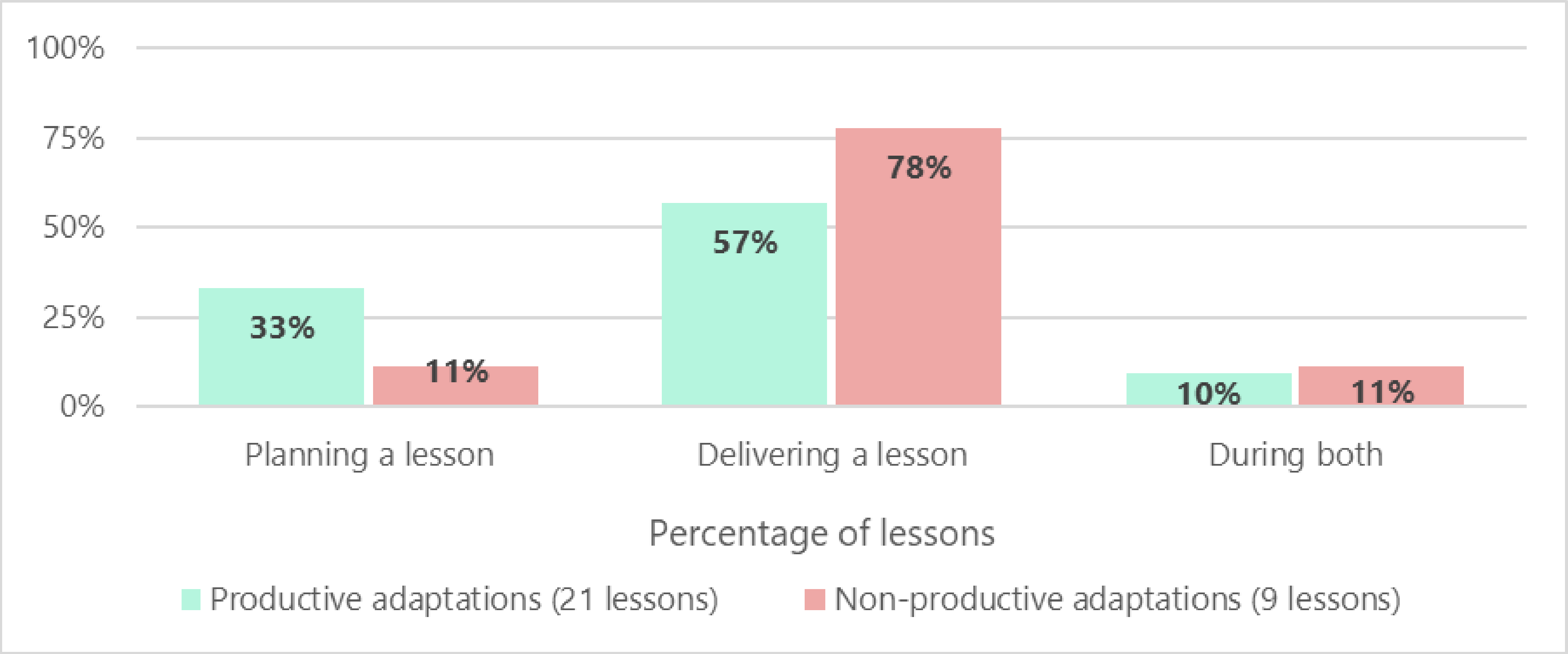
4. Teachers who made productive adaptations more often indicated they **considered learning progressions** while teachers who made nonproductive adaptations **did not indicate they did so**.

Finding 1: Teachers made productive adaptations more often than nonproductive ones.



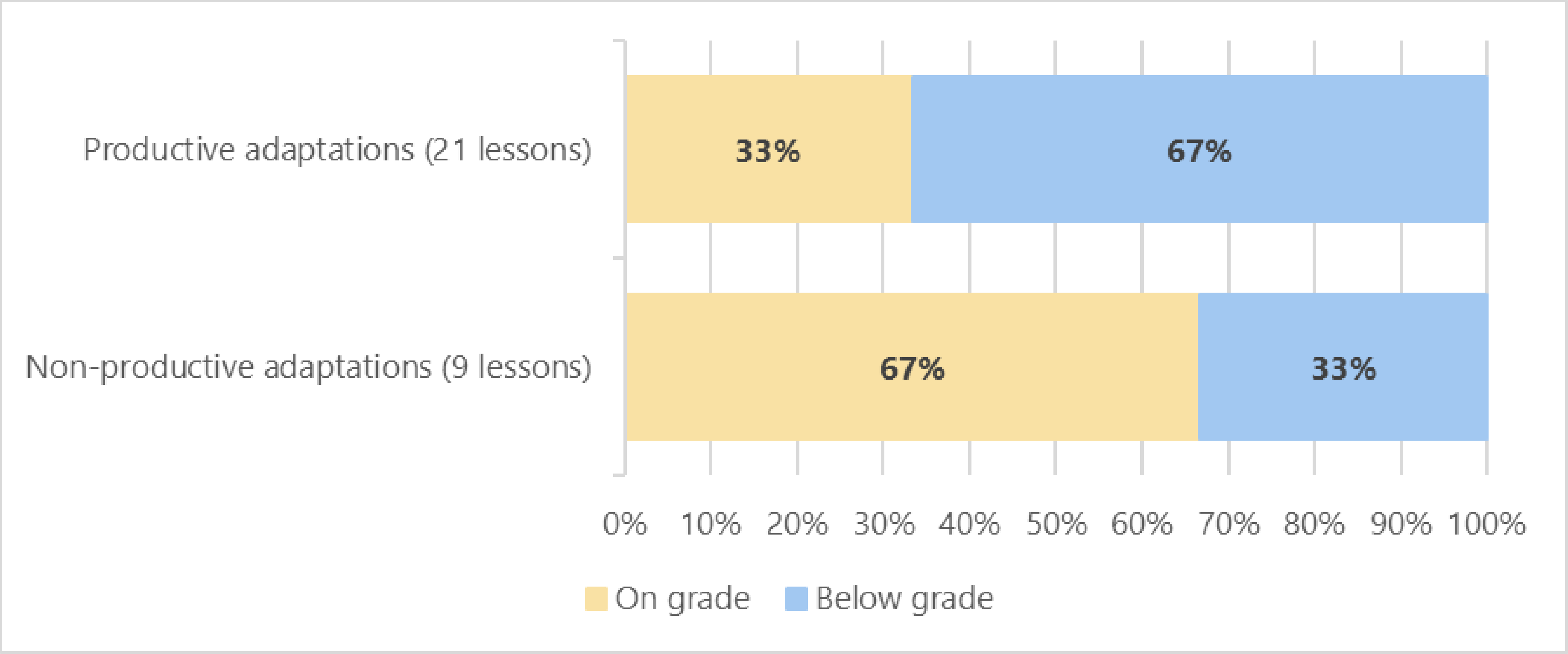
Data source: classroom observations during the 2021-22 and 2022-23 school years; lesson plans or instructional materials; and curriculum materials

Finding 2: Teachers made productive adaptations more often while planning a lesson and nonproductive adaptations more often while delivering a lesson.



Data source: classroom observations during the 2021-22 and 2022-23 school years; lesson plans or instructional materials; and curriculum materials

Finding 3: Teachers made productive adaptations more often to content below grade level and made nonproductive adaptions more often to content on grade level.



Data source: classroom observations during the 2021-22 and 2022-23 school years; lesson plans or instructional materials; and curriculum materials

Finding 4: Teachers who made productive adaptations more often indicated they considered learning progressions while teachers who made non-productive adaptations did not indicate they did so.

Why is the lesson productive?

6.NS.B.2 and **5.NBT.B.6** fall on the *Division and Multiplication* learning progression.[a]

(Intended and planned standard: 6.NS.B.2, n.a.[b])

"*I can't teach them how to divide using decimals—which is what the goal is—if they don't know how to do long division. So that's why I focused on problems they were supposed to learn in fifth grade, but most of them didn't. So that's something we were aware of.****"***

(Delivered standards: 5.NBT.B.6)

"I can't teach them how to divide using decimals—**which is what the goal is**—if they don't know how to do long division. So that's why I focused on problems they were **supposed to learn in fifth grade**, but most of them didn't."

Data source: interviews conducted with teachers after one classroom observation per teacher during the 2021-22 and 2022-23 school years

Finding 4: Teachers who made productive adaptations more often indicated they considered learning progressions while teachers who made non-productive adaptations did not indicate they did so.

Why is the lesson nonproductive?

8.EE.A.2 falls on the *Linear and Simultaneous Functions* learning progression.[a] **7.G.B.4** falls on the *Length, Area, and Volume* learning progression.[a]

(Intended and planned standard: 8.EE.A.2)

"*In chapter seven, [we're supposed to cover] squares and square roots and then cube and cube roots and then the Pythagorean theorem. But I like to talk about the geometry application first because I feel that's what this particular class likes.*"

(Delivered standards: 7.G.B.4)

"In chapter seven, [we're supposed to cover] squares and square roots and then cube and cube roots **and then the Pythagorean Theorem**. But **I like to talk about the geometry application first** because I feel that's what this particular class likes."

Data source: interviews conducted with teachers after one classroom observation per teacher during the 2021-22 and 2022-23 school years

What might be driving these trends?

- We found **no differences** between curricula teachers are using.
  - We considered individual lessons and standards rather than the curriculum as a whole. The standards alignment of an individual lesson is not dependent on whether the curriculum as a whole is standards aligned.
- We found **no patterns** by teacher demographics or by grade, district, or content being taught.
- We found that teachers who made nonproductive adaptations had, on average, a **higher educational degree**, an **advanced teaching certificate**, and **more teaching experience**.

This implies…

1. Districts might consider offering **professional learning** on using math learning progressions to productively adapt their curricula.

2. Curricular adaptations should focus on meeting the needs of the students while maintaining the learning progressions within **high-quality, standards-aligned materials**.

3. Teachers may need additional class time to **prioritize foundational skills** that build toward the intended standards or may need to provide additional support to students outside of general education classes.

4. Teachers should consider how and when to adapt lessons to include **above-grade-level content**.

Limitations

- We focused on **content adaptations** rather than other adaptations (for example, translating content for English Learners)
- Findings **may not be generalizable** due to small sample size (37 teachers; 85 classroom observations)
- We **could not determine consistency** due to observing one or two lessons per teacher
- We **could not determine if adaptations were needed** since we did not collect formative or summative student assessment data

Additional NCTM session to join

We are developing an educator practice guide that seeks to:

- **Define "productive adaption"** as a change to instructional material that maintains alignment along the mathematical learning progression, so student learning is moving toward the intended goal.
- **Build teacher capacity to make productive** adaptations to instructional units, including changes to their instructional materials, instructional strategies, or student performance tasks.
- **Provide examples of productive adaptations** teachers can make when students perform below or above grade level.

Session 163 – *Meeting students where they are: How to productively adapt instructional material*. **Today from 1:00 – 2:15; Hyatt; Dusable**

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