Mathematica
Progress Together

Analyzing Middle School Mathematics Curricula

A Comparative Study Using Three Measurement Tools

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Riley Stone, Lena Rosendahl, Amy Johnson, and Barbara Harris

This report was prepared for the Bill & Melinda Gates Foundation. The findings and conclusions contained within are those of the authors and do not necessarily reflect positions or policies of the Bill & Melinda Gates Foundation.

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Contents

Exhibits

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I. Introduction

The Analysis of Middle School Math Systems (AMS) project is part of a larger set of investments by the Bill & Melinda Gates Foundation intended to help students who are Black, Latino, multilingual learners, and/or experiencing poverty succeed in mathematics. Pivotal to this success is the use of high-quality mathematics curricula and aligned professional learning. Defining dimensions of curricular quality and using them to assess instructional materials is a critical step in determining whether teachers have high-quality curricula.

A core hypothesis guiding the AMS project is that teachers' use of high-quality mathematics curricula matters (Cobb & Jackson, 2011; Elmore et al., 2014), particularly when curriculum-aligned, culturally responsive professional learning (PL) supports use of the materials. When that is the case, we assume that teachers will draw from the curricula to provide a better classroom experience for students who are Black, Latino, multilingual learners, and/or experiencing poverty. Research has demonstrated that high-quality, standards-based curricula and professional learning can improve a teacher's ability to implement more ambitious and inclusive instructional practices that incorporate diverse learners' mathematical knowledge bases (including students' mathematical thinking and their cultural, linguistic, and community-based knowledge) into classroom instruction (Desimone & Garet, 2015; Moyer et al., 2011; Tarr et al., 2008; Turner et al., 2012). In Exhibit I.1, we display the hypotheses that guided the development of the AMS project's research questions.

Exhibit I.1. Hypotheses guiding AMS study research questions

If	Then
• Teachers use high-quality mathematics curricula that are embedded in coherent instructional contexts • Teachers receive high-quality professional learning support that aligns with the intended curricula and develops their mathematical knowledge for culturally responsive mathematics teaching	• Teachers will plan lessons from the curricula that align with standards, are cognitively demanding, are culturally responsive, and support students' mathematics language development and language diversity; • Teachers will enact curricula with integrity and make productive adaptations; • Teachers' beliefs and instructional capacity will improve; and then • Students who are Black, Latino, multilingual learners, and/or experiencing poverty will have a better classroom experience in terms of their mathematics enjoyment, achievement identity, performance, persistence, self-efficacy, and growth mindset.

The education field invests billions of dollars annually in instructional materials (NCES 2021) that must meet the needs of diverse school systems across the country (Aydin et al., 2017). Given the magnitude of the investment and the crucial influence of curricula on student outcomes (Squires, 2012; Parrish & Bryd, 2022; Whitehurst, 2009; Agodini et al., 2013; Agodini & Harris, 2010), it is imperative that districts and schools know which materials are considered high quality. We define high quality curricula as curricula that are aligned to state or national standards, useable, and supportive of culturally responsive instructional delivery.

This report seeks to understand how curricula vary along the dimensions of high quality. Given our focus on understanding the mathematics classroom experiences of students who are Black, Latino, multilingual learners, and/or experiencing poverty, along with recent attention directed to the importance of CRMT, we pay particular attention to how curricula provide guidance to enact CRMT practices.

II. Methods

For this analysis, we assessed each of the study's six curricula by using three tools that differentially measure quality across domains. The six curricula[1] include the following:

- **Into Math.** Into Math is a grade K–8 core curriculum, published by Houghton Mifflin Harcourt (HMH), that first emphasizes conceptual understanding and reasoning skills before connecting students' understanding to procedural practice in concepts and skills. Many lessons span several days, helping build perseverance and allowing teachers to emphasize the importance of productive struggle.
- **Eureka Math.** Also known as EngageNY, Eureka Math is a grade pre-K (PK)–12 core curriculum, published by Great Minds, that sequences mathematical progressions to present high-quality mathematics content, teaches persistence in problem solving, and prepares students to understand advanced mathematics. The curriculum emphasizes mathematical thinking, problem solving, and conceptual understanding by teaching students to use numerous strategies for solving difficult problems.
- **Illustrative Mathematics.** Illustrative Mathematics is a grade K–12 problem-based, core curriculum, published by three publishers[2] (we reviewed the version published by LearnZillion/Imagine Learning). The curriculum uses a "learn by doing" approach and emphasizes solving problems in mathematical and real-world contexts and constructing arguments by using precise language. Teachers facilitate student learning with research-based mathematical routines to guide learners to understand and make connections between concepts and procedures.
- **Big Ideas Math.** Big Ideas Math is a grade K–12 core curriculum, published by Big Ideas Learning, that emphasizes focus, coherence, and rigor. The curriculum emphasizes focus by combining direct instruction with inquiry activities to help students move from concrete to abstract understanding and to learn to apply mathematical concepts in real life. Its approach to coherence involves designing each grade level to build on the concepts taught in the earlier grades, with minimum repetition. The curriculum emphasizes rigor by incorporating attention to conceptual understanding into each lesson as well as by prioritizing procedural fluency and application of mathematics to real-life problems.
- **California Glencoe Math (California Math).** California Math is a grade 6–8 core curriculum, published by McGraw Hill Education, that embeds conceptual understanding, application, and procedural fluency in each lesson and assessment. Each lesson includes opportunities to apply mathematical concepts in real life.
- **Key Elements of Success (KEMS).** KEMS is a grade 3–8 core curriculum, developed by National Training Network, that emphasizes several representations of mathematics content, structured scaffolding of problem-solving techniques, student cooperative learning, and connections among concrete, pictorial, verbal, and abstract representations of mathematics content.

We used three tools to measure the quality of the curricula based on their (1) usability, (2) alignment to the Common Core State Standards (CCSS), and (3) prevalence of guidance for teachers to enact the curriculum in culturally responsive ways. These tools[3] include the following:

- **EdReports.** EdReports issues a "meets, partially meets, or does not meet expectations" rating that is color coded as green, yellow, and red, respectively. Ratings are determined by how much

[1] See Appendix A for summaries of each curriculum.
[2] Kendall Hunt, LearnZillion/Imagine Learning, and McGraw Hill all publish Illustrative Mathematics.
[3] See Appendix B for a more detailed description of each tool.

instructional materials align with grade-level learning and mathematical practice standards, facilitate student learning, and enhance a teacher's ability to differentiate and build knowledge within the classroom. Educator-led review teams review the curricula independently and then discuss evidence as a team to issue the ratings. We compiled ratings from the publicly available EdReports Report Database for the study curricula. Scores represent the full middle school curriculum (grade 6–8). Three of our six study curricula earned a "green" rating from EdReports; two others earned a non-green rating, and one curriculum was not rated. We signal green- and non-green– rated curricula through shading in the exhibits in this report.

- **The Surveys of Enacted Curriculum (SEC).** The SEC assesses the extent to which a curriculum's topic emphasis and cognitive demand of students' performance expectations align with the CCSS recommendations. A team of mathematics education experts with the Center for Curriculum Analysis (CCA) used the SEC to score the study curricula. Scores represent the full middle school curriculum (grade 6–8). The ratings are not yet public, but the SEC plans to include overall alignment scores and content maps on its website, potentially within in the next six months.[4]

- **The Culturally Responsive Mathematics Teaching (CRMT) curriculum coding tool.** The CRMT curriculum coding tool measures the prevalence of a curriculum's guidance for enacting culturally responsive instruction, such as connecting content to student culture and identities, providing all students with rigorous material,[5] and attending to the power and participation of students throughout the learning process. The AMS team developed the tool by using an adaptation of the Culturally Responsive Mathematics Teaching (CRMT) Lesson Analysis Tool (Aguirre & del Rosario Zavala, forthcoming).[6] A team of trained Mathematica staff independently reviewed the teacher's guide for each curriculum and noted evidence or opportunities to enact CRMT within nine domains (see callout box for the domains). After coding, the review team came to consensus on discrepancies in the codes before finalizing the data. Unlike EdReports and the SEC—which issue ratings based on the full grade 6–8 curriculum—the CRMT review team reviewed a sample of grade 6 lessons to issue CRMT scores.[7] The scores are not publicly available.

Nine domains of the CRMT curriculum coding tool

1. Community and Cultural Funds of Knowledge
2. (Re) Humanizing
3. Student Ideas and Thinking
4. Cognitive Demand
5. Maintaining Rigor
6. Affirming Multilingualism
7. Distributing Intellectual Authority
8. Disrupting Power
9. Taking

Note: See Appendix B for descriptions of each domain.

In Exhibit II.1, we list the categories of curricular quality assessed by each tool and briefly describes the category.

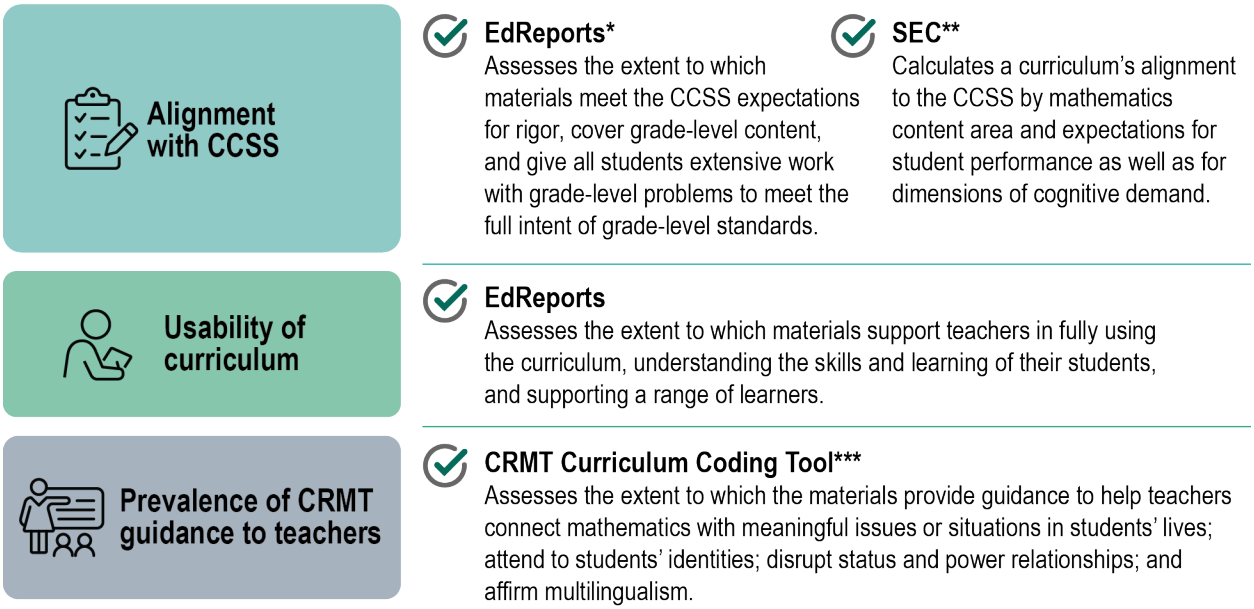
[4] Detailed results will be available only for those with a subscription. Overall alignment scores appear in Exhibit C.1 and content maps for each curriculum and the CCSS in Exhibit C.6.

[5] Even though the inclusion of rigorous material is not unique to CRMT, our CRMT curriculum coding tool uniquely measures when guidance exists to ensure ALL students have access to rigorous material. EdReports and the SEC measure whether more rigorous material exists, but they do not indicate who should access each task.

[6] The tool was adapted because the original tool was designed for use in dialogue with teachers rather than as a means of quantitatively scoring materials.

[7] We piloted this tool with six to nine lessons in each of the four units selected from each curriculum. We recommend further validation before applying the tool to an entire curriculum.

Exhibit II.1. Crosswalk of each tool's measurement categories



* Available to the public.

** Overall alignment and content maps may become public in the next six months.

*** Not available to the public.

We used descriptive analyses of the data from each tool to examine the extent to which the curricula are usable, align with the CCSS, and include guidance for enacting CRMT practices. In summary,[8] we found that:

- **Illustrative Math, Into Math, and Eureka Math meet expectations for alignment with the CCSS grade-level and mathematical practice expectations but differ in their usability, as measured by EdReports.** Of the three curricula, Illustrative Math ranks the highest, and Eureka Math ranks the lowest in Usability.
- **All six curricula place greater emphasis than the CCSS recommends on procedural practice**, as measured by the SEC, resulting in less emphasis than recommended on more cognitively demanding tasks**.**
- **All six curricula also place greater emphasis than the CCSS recommends on Operations, Data Displays, and Measurement topics**, as measured by the SEC, resulting in less emphasis on Algebra, Probability, and Statistics topics.
- **All six curricula offer limited explicit guidance in enacting CRMT practices.** The curricula provide more meaningful guidance in traditional areas of instructional reform, such as cognitive demand, student ideas and thinking, and distributing intellectual authority, than in domains related to students' identities, power, and participation.
- **Illustrative Math is almost always the highest-rated curriculum, across each tool and its domains, while KEMS is almost always the lowest-rated curriculum.**

[8] Appendix C provides in-depth results for each tool. Appendix D provides in-depth results by curriculum.

Looking across our findings, we conclude the following about how findings may be useful to curriculum developers, educators, and other decision makers within the field of mathematics education:

- **Curriculum designers face a difficult balancing act when determining the relative emphasis assigned to topics**. Closer alignment of curricula to the CCSS standards and improved guidance for CRMT practices may better support teachers in engaging students in mathematics.
- **Districts need information sources that provide a more comprehensive assessment of curricula**.
- **Targeted professional learning is an important way to help teachers bridge the gaps between selected curricula and a district's strategic vision and priorities**.

In the following sections, we detail our findings, implications, and next steps for interested parties.

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III. Findings

Illustrative Math, Into Math, and Eureka Math meet expectations for alignment with the CCSS grade-level and mathematical practice expectations but differ in their usability, as measured by EdReports.

EdReports measurement domains

EdReports is designed to issue a meets (green), partially meets (yellow), or does not meet expectations (red) rating based on a curriculum's performance across three domains.

- **Focus and Coherence:** Do the materials assess grade-level content, and are they coherent and consistent with the CCSS?
- **Rigor and Mathematical Practices:** Do the materials meet the CCSS expectations for rigor and mathematical practices?
- **Usability:** Do the materials support teachers in fully using the curriculum, understanding students' skills and learning, and supporting a range of learners?

The three green-rated study curricula all received the highest rating possible in EdReports Focus and Coherence domain but diverge in the Rigor and Mathematical Practices domain and even more in the Usability domain (see callout box for the three domains EdReports measures). Of the three study curricula rated green, Illustrative Math ranks the highest and Eureka Math the lowest for Usability.

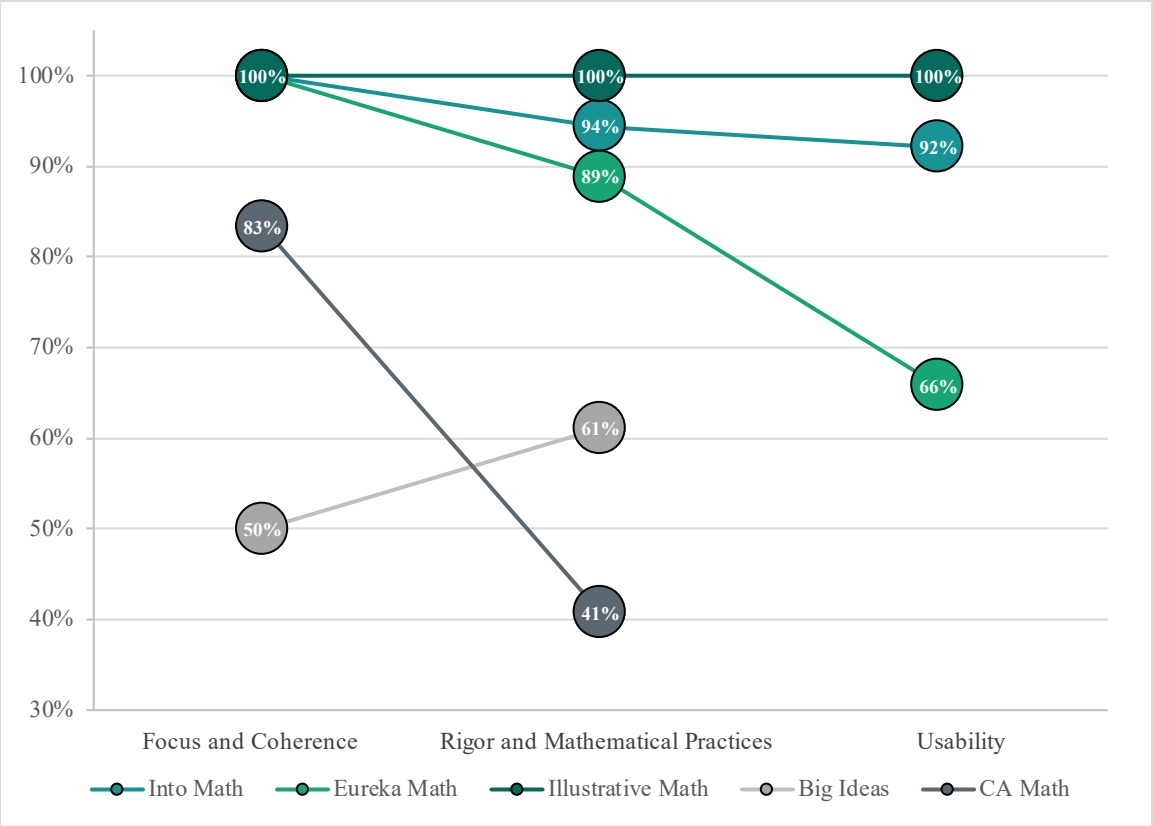
For Rigor and Mathematical Practice, Illustrative Math scored 100 percent,[9] Into Math scored 94 percent, and Eureka Math scored 89 percent. The non-green curricula scored lower in Focus and Coherence and Rigor and Mathematical Practices. Big Ideas scored 83 percent in Focus and Coherence and 41 percent in Rigor and Mathematical Practices while California Math scored 50 and 61 percent, respectively.

In Exhibit III.1, we present each curriculum's EdReports scores as percentages across the three domains measured by the tool.[10] EdReports reviewers apply the three domains sequentially. Only materials that meet expectations for the Focus and Coherence and Rigor and Mathematical Practices domains are assessed on the Usability domain. (In other words, the reviewers do not score non-green curricula on the Usability domain.)

[9] To calculate percentages, we summed each grade band's score and divided by the sum of total possible points across grade bands. Generally, green ratings are those above 85 percent, yellow ratings are those between 60 and 85 percent, and red ratings are those below 60 percent.

[10] EdReports reviewers applied the three domains sequentially; thus, only materials that meet expectations for Focus and Coherence and Rigor and Mathematical Practices are assed on the Usability domain. (In other words, non-green curricula are not scored on the Usability domain.)

Exhibit III.1. EdReports ratings by curriculum



Note: EdReports has not rated KEMS. Big Ideas and California Math have no Usability ratings because they were non-green for Focus and Coherence and Rigor and Mathematical Practices. Non-green curricula are not scored on the Usability domain.

Green curricula primarily diverge in their Usability domain scores. Illustrative Math and Into Math scored above 90 percent for Usability (100 and 92 percent, respectively), and Eureka Math scored 66 percent. To provide additional qualitative data on teacher views of the usability of the study curricula, we synthesized survey and focus group data from CUrriculum RAtings by TEachers (CURATE) (CURATE, 2023).[11]

- **Illustrative Math.** Teachers indicated that the materials present coherent learning progressions, consistent lesson structures, and strong support for instructional routines. At the same time, however, teachers sensed that pacing may be difficult.
- **Into Math.** Teachers indicated that the materials specify clear objectives for student learning, offer suggestions for grouping strategies to support differentiated instruction, and include a breadth of informal and formal assessments. Teachers also noted that pacing is reasonable and flexible. However, the assessments fail to make recommendations for teacher follow-up per the assessment results, and the materials lack content to build teachers' mathematical content knowledge.
- **Eureka Math.** Teachers indicated that the materials advance student learning, build teachers' content knowledge, and provide consistent lesson structure. Nonetheless, teachers reported that pacing can be difficult, that the curriculum does not allow adequate time for reteaching when necessary, and that the

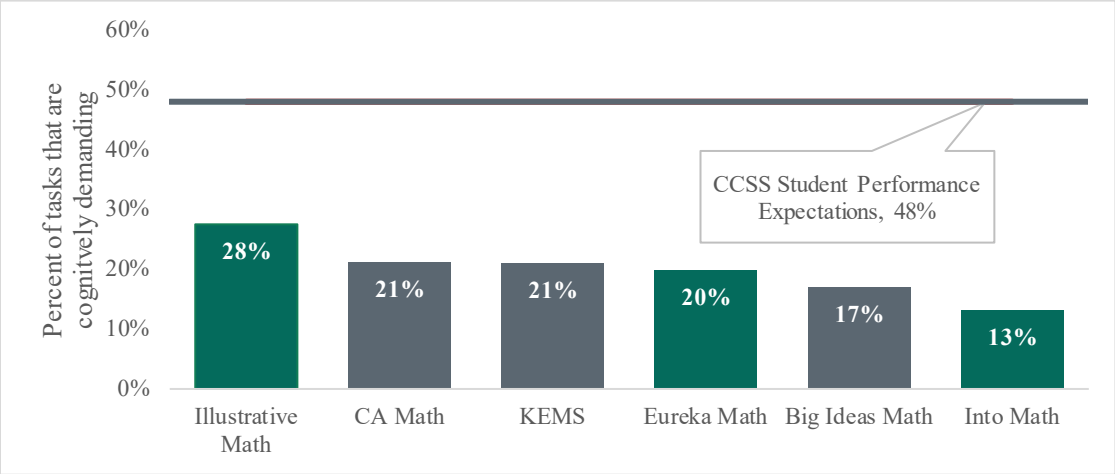
[11] Focus group data for non-green curricula are not available.

guidance is weak for assessing existing knowledge and for suggesting follow-up for students struggling with the material.

All six curricula place greater emphasis than the CCSS recommends on procedural practice, as measured by the SEC, resulting in less emphasis than recommended on more cognitively demanding tasks.

On average, 20 percent of the study curricula material emphasizes cognitively demanding tasks, which is less than half of the percentage recommended by the CCSS (48 percent). Of the six curricula, Illustrative Math places the greatest emphasis (28 percent) and Into Math the least emphasis (13 percent) on cognitively demanding tasks (Exhibit III.2). [12]

Exhibit III.2. Average amount of cognitively demanding tasks in curricula, SEC



All six curricula also place greater emphasis than the CCSS recommends on Operations, Data Displays, and Measurement topics, as measured by the SEC, resulting in less emphasis on Algebra, Probability, and Statistics topics. [13]

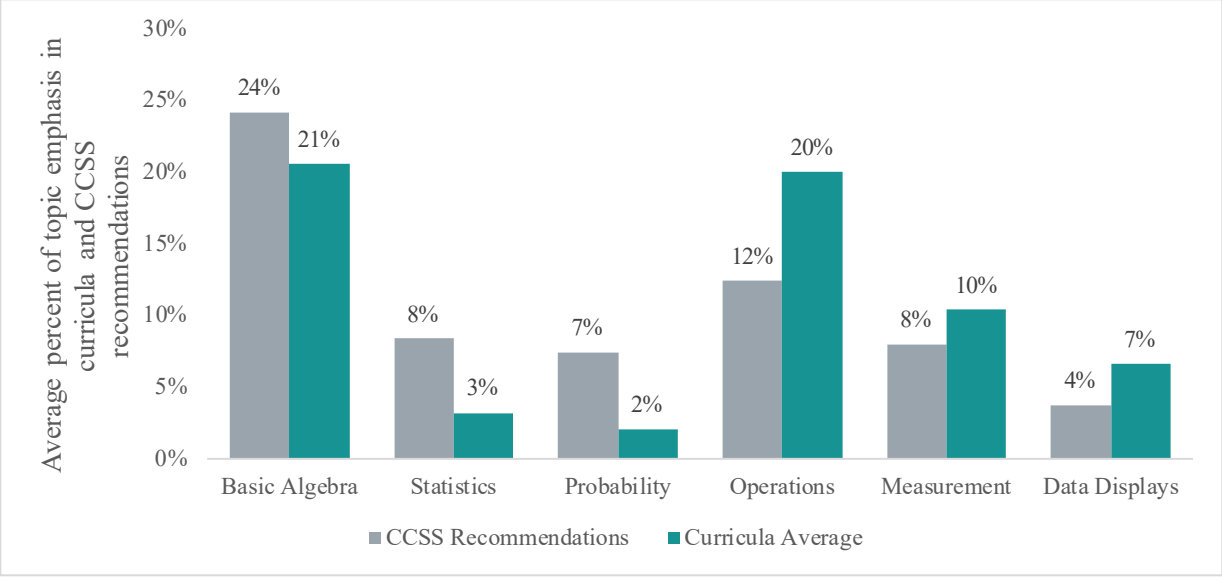
The CCSS recommends that curricula place about 24 percent of topic emphasis on Basic Algebra, 8 percent on Statistics, and 7 percent on Probability. The six study curricula place, on average, 21 percent emphasis on Basic Algebra, 3 percent on Statistics, and 2 percent on Probability. In place of these topics, study curricula place greater emphasis on Operations, Measurement, and Data Displays than recommended by the CCSS (Exhibit III.3). The CCSS recommends a 12, 8, and 4 percent emphasis on each of these topics, respectively; on average, curricula place a 20, 10, and 7 percent emphasis on each,

[12] The SEC defines cognitively demanding tasks as those requiring students to demonstrate understanding; to conjecture, generalize, and prove; or to solve nonroutine problems and make connections. These tasks require students to exercise their knowledge of mathematics, think critically or creatively, and demonstrate their understanding of how to apply less cognitively demanding tasks, such as recalling information, following directions, or performing procedures, to a more complex problem.

[13] The SEC measures the relative emphasis that each curriculum places on different content topics in terms of percentages. These percentages, across topics, sum to 100 percent of the total curriculum and roughly describe students' relative exposure to each topic within a curriculum if that curriculum is implemented exactly as written. Placing greater emphasis on one topic, such as Operations or Measurement, leads to less emphasis on other topics, such as Algebra or Probability.

respectively. These levels of emphasis suggest that the curricula prioritize more foundational topics that students must master before they engage with more advanced topics.

Exhibit III.3. Comparison between average percentage emphasis placed on selected mathematical topics in grade 6–8 curricula and CCSS recommendations



All six curricula offer limited explicit guidance in enacting CRMT practices.

On average, the study curricula received a rating of about 2 (out of 5) on the CRMT curriculum coding tool. In other words, on average, the curricula provide at least one brief instruction (for example, to a subset of students or during a short task such as a warm-up) to address CRMT practices in each lesson. Illustrative Math had the highest average score across CRMT domains, at 2.69 (Exhibit III.4), suggesting that the CRMT instruction(s) in Illustrative Math lessons usually include all students in the class and take place during the main activities in a lesson.

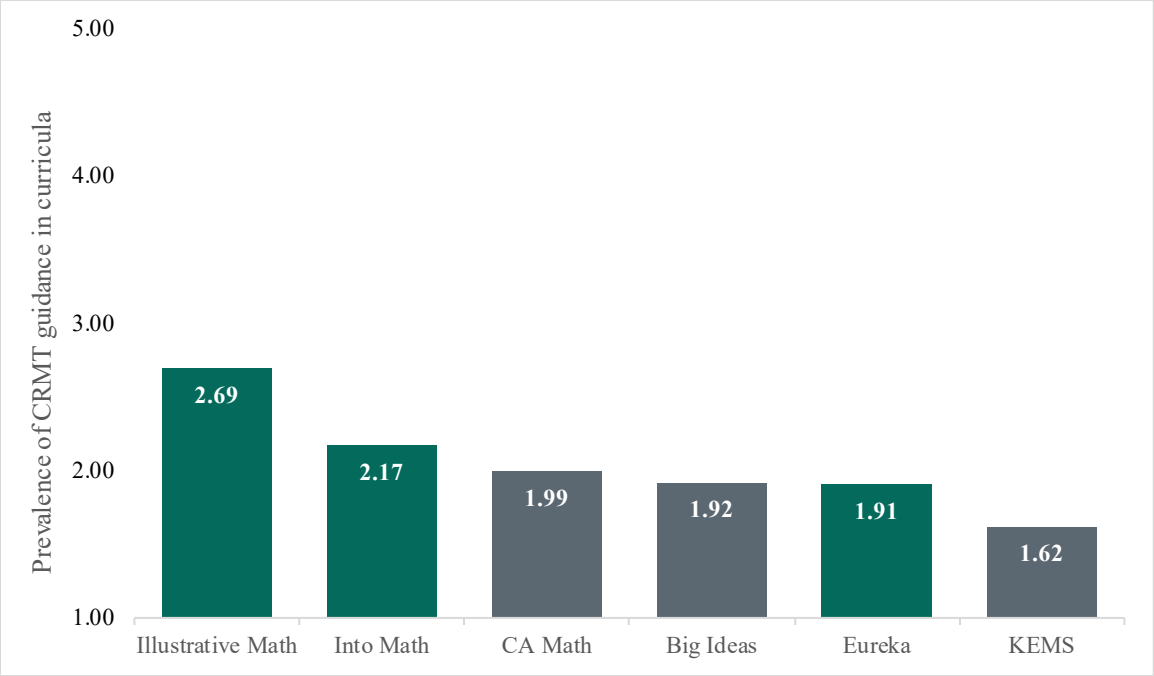
No curriculum achieved an average rating of 3 or higher, meaning that no meaningful instructions addressed CRMT practices in every lesson of the curriculum.

CRMT curriculum coding tool scores

CRMT ratings reflect the prevalence of guidance within the curriculum for how teachers can adopt CRMT practices. The tool uses a scale of 1 to 5 to represent the following:

- A score of 1 represents no instruction or guidance across lessons within a domain.
- A score of 2 represents a brief instruction or guidance that may not include all students.
- A score of 3 represents a meaningful instruction or guidance for all students.
- A score of 4 represents several meaningful instructions, with at least one including all students.
- A score of 5 represents an entire lesson with guidance pertaining to the relevant domain.

Exhibit III.4. Prevalence of CRMT guidance in curricula, CRMT curriculum coding tool

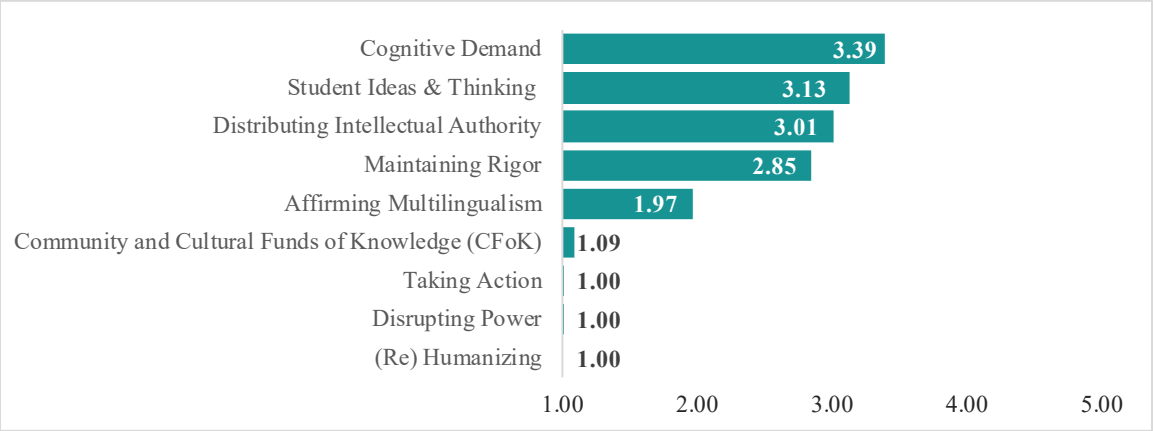


When looking across the nine domains measured by the CRMT curriculum tool, we find that the curricula provide more meaningful guidance to teachers in more traditional areas of instructional reform, such as Cognitive Demand, Student Ideas and Thinking, and Distributing Intellectual Authority (Exhibit III.5). These CRMT areas emphasize student understanding of underlying concepts, patterns, and properties; communication; and ownership.[14] In CRMT domains related to students' identities, power, and participation, the curricula provide little or no guidance. For example, the curricula scored at or near 1, on average, for Community and Cultural Funds of Knowledge, (Re) Humanizing, Disrupting Power, and Taking Action. This means that the curricula provide almost no instructions for teachers to celebrate diversity in students' culture and knowledge and thus do not empower students to build positive mathematics identities and apply their skills to enact positive change.[15]

[14] We acknowledge that these areas are traditional areas for curriculum reform. To incorporate a culturally responsive focus, we applied an equity lens to the CRMT curriculum coding tool. For example, for Cognitive Demand, the tool did not solely measure the extent to which a lesson is cognitively demanding but rather the extent to which the lesson ensures that all students have access to cognitively demanding tasks.

[15] See Appendix B for detailed descriptions of each CRMT curriculum coding tool domain.

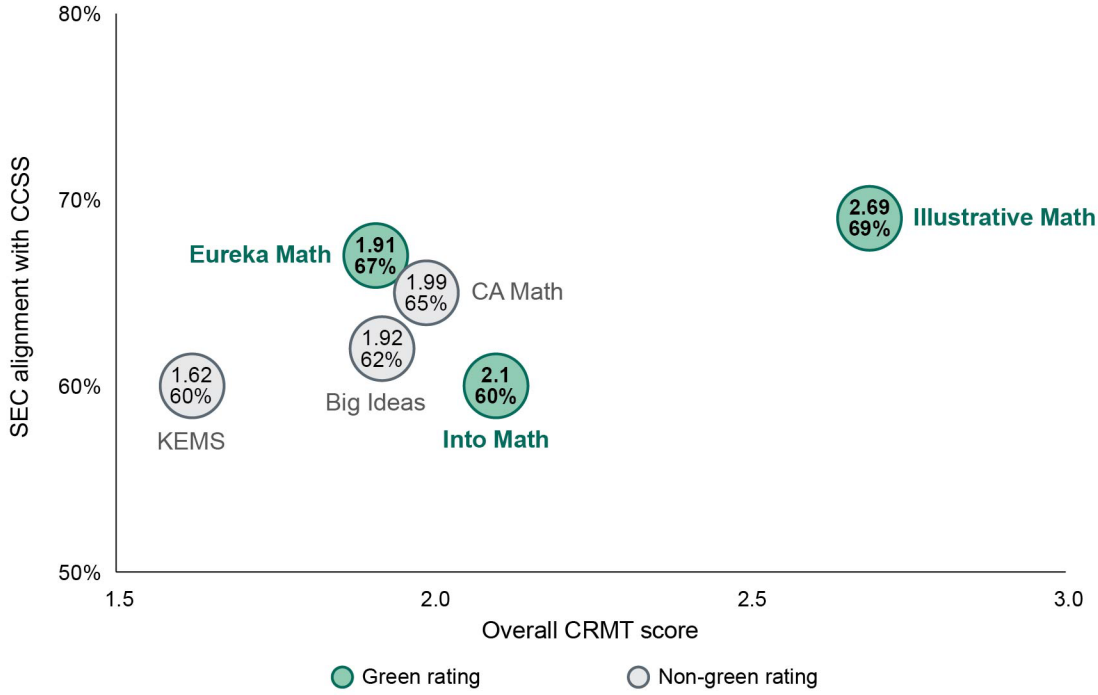
Exhibit III.5. Average prevalence of CRMT guidance in curricula by domain, CRMT curriculum coding tool



Illustrative Math is almost always the highest-rated curriculum, across each tool and its domains, while KEMS is almost always the lowest-rated curriculum.

In Exhibit III.6, we present a composite view of how curricula perform across all three tools: EdReports, the SEC, and the CRMT curriculum coding tool. Curricula rated green by EdReports appear as green dots, and the x- and y-axes show average ratings by the CRMT tool and the SEC, respectively. Illustrative Math had the highest scores across all three tools, while KEMS had consistently low scores across the tools, exhibiting the lowest average score on the CRMT tool and tied with Into Math for the lowest overall alignment score. (EdReports has not rated KEMS.)

Exhibit III.6. Curricular average CRMT score versus SEC alignment with CCSS

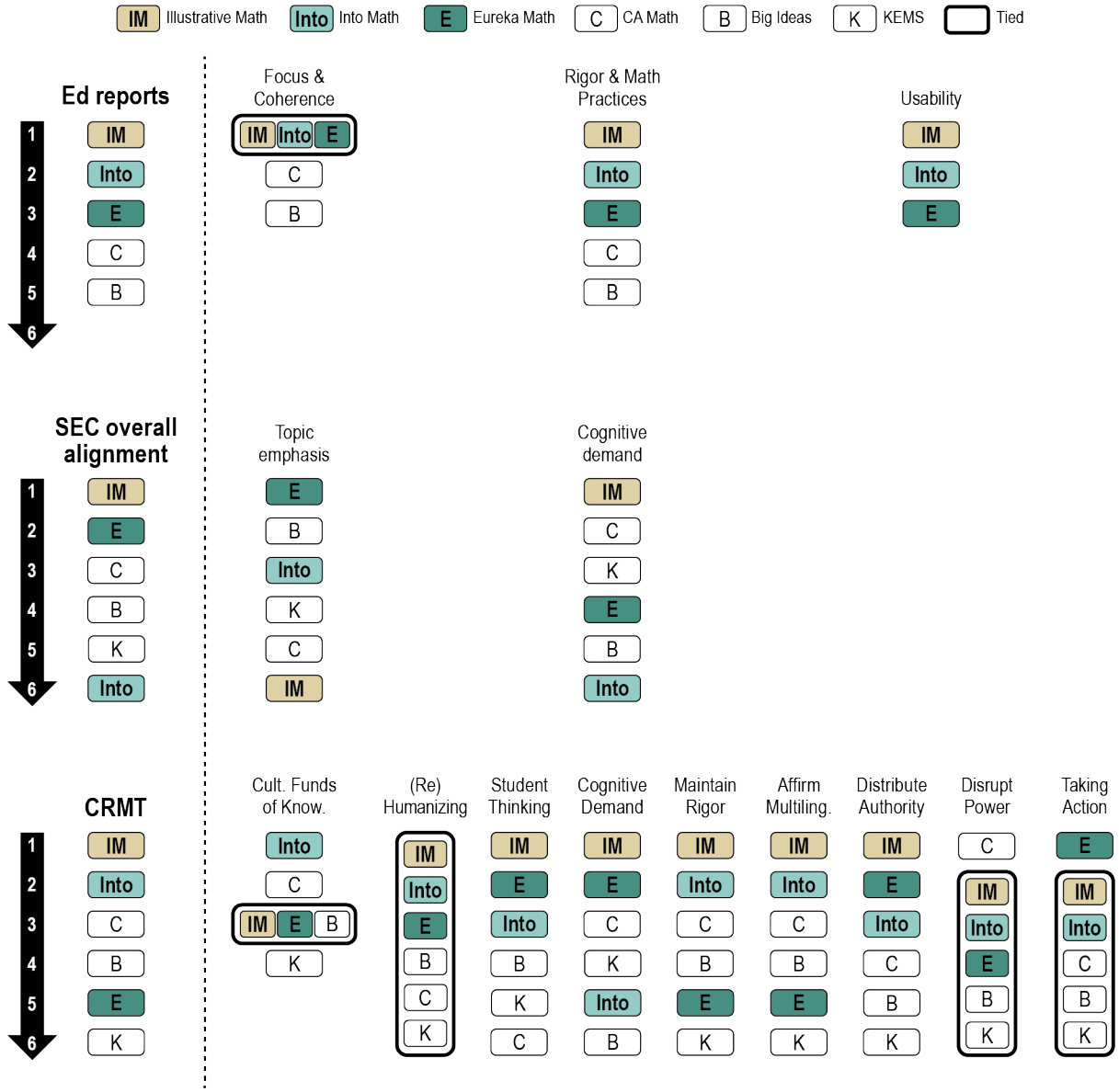


Note: The SEC alignment scale ranges from 0 to 100 percent; the CRMT curriculum coding tool scale ranges from 1 to 5.

In Exhibit III.7, we present a more detailed view of how the curricula ranked across each tool. The bolded column on the left represents performance on each tool overall, while the nonbolded columns to the right provide performance ranks across each tool's domains. Below the headers, each row represents a curriculum's rank from first to sixth. Ties between curricula appear in a bolded box. To illustrate:

- **Illustrative Math** ranks first across all of the tools overall scores, but has the weakest topic emphasis alignment, according to the SEC. For the CRMT curriculum coding tool, Illustrative Math outperforms the other curricula in five of nine domains and performs similarly to the other curricula in the other four domains.
- **Into Math** ranks second overall on EdReports and the CRMT curriculum coding tool but last on overall alignment to the CCSS according to the SEC. Its low overall score in the SEC domains is mainly attributable to its comparatively lower score for cognitive demand alignment.
- **Eureka Math** ranks fifth on the CRMT curriculum coding tool but second on overall alignment to the CCSS according to the SEC. Its relatively high overall alignment score in the SEC domains is mainly attributable to its strong topic emphasis alignment (first). It ranks fourth in cognitive demand alignment.

Exhibit III.7. Relative curriculum ratings on EdReports, SEC, and CRMT curriculum coding tool



Notes: Black boxes indicate ties within a domain. For example, Illustrative Math, Eureka, and Big Ideas all ranked third in the Community and Cultural Funds of Knowledge domain. For the (Re) Humanizing, Disrupting Power, and Taking Action domains, the curricula with tied scores did not provide any guidance to enact these CRMT practices. Big Ideas and California Math are not rated on the Usability domain because they do not meet expectations for the first two sets of ratings. EdReports has not rated KEMS. Exhibit C.1. presents the scorers associated with these ranks, and Appendix D displays each curriculum's scores across the three tools in Exhibit 9.

IV. Implications and next steps

The analyses in this report include a robust sample of data to capture the prevalence of characteristics in the six study curricula—specifically, the extent to which curricula are usable, standards-aligned, and culturally responsive. Below, we summarize the implications from our findings.

Implications

Curriculum designers face a difficult balancing act when determining the relative emphasis assigned to topics. Closer alignment of curricula to the CCSS standards and improved guidance for CRMT practices may better support teachers in engaging students in mathematics.

Our analysis found that the study curricula placed more emphasis than the CCSS recommends on tasks focused on performing procedures; as a consequence, the curricula include less content focused on cognitively demanding tasks. The SEC identifies cognitively demanding tasks as those requiring students to apply their foundational skills and knowledge to think critically or creatively in solving more complex, realistic problems. However, performing mathematical procedures builds fluency. A key issue for curriculum developers is how to find the ideal balance between students' exercise of lower cognitive demand tasks to build foundational skills and students' application of those skills to think critically or creatively while performing more cognitively demanding tasks.

Measurement challenge

EdReports found high levels of alignment with CCSS content and mathematical practice expectations, but the SEC found some misalignment. Given that these tools rely on different measures at different levels of granularity, the differences in findings are not surprising. The SEC tool allows for greater differentiation, leading to the ability to identify nuances across alignment. Educators trying to synthesize information across both tools may struggle to reconcile differences in alignment.

Increasing relative emphasis on any one topic means decreasing relative emphasis on others. Our results show that, in general, curricula designed with a strong emphasis on foundational topics (for example, Operations and Measurement) that support more advanced topics (for example, Algebra, Statistics, and Probability) means less emphasis on those advanced topics. Introducing these topics in middle school is critical to preparing students for high school coursework (Klute, 2021; Watts et al., 2014).

Curricula also place a strong emphasis on data displays which involves creating and interpreting visual representations of data and is an important topic for building data literacy and mathematical intuition. However, understanding data displays requires an understanding of statistics and probability concepts. Given that our analysis shows a lack of emphasis on these more advanced topics in middle school, compared to the emphasis recommended by the CCSS, students may struggle with grade-level course work in high school and may not benefit as intended from topics such as Data Displays, despite a strong effort by curriculum developers to ensure that it receives thorough coverage (Bargagliotti, 2022; Bowen, 2021).

It is also worth noting that, even though topic emphasis and relative cognitive demand within curricula are important, the actual topic emphasis and cognitive demand that students experience within a classroom is significantly influenced by how teachers choose to implement and adapt curricula. For

example, a curriculum may place greater emphasis on operations and lower cognitive demand tasks, but teachers may choose to shift the relative emphasis to other tasks and better align student experiences with the CCSS recommendations. Alternatively, if a curriculum places relatively low emphasis on statistics and introduces statistics late in the year, teachers may not have the time needed to address the material.

Curricula provide teachers with limited guidance for enacting CRMT practices**.** Improving CRMT guidance can help teachers adopt practices that build students' mathematics identities and support students' ability to engage with mathematical topics through their Community and Cultural Funds of Knowledge. Although CRMT is a relatively new field and still lacks evidence on how best to use CRMT practices to improve student outcomes, many CRMT domains, such as engaging students' Community and Cultural Funds of Knowledge, reflect practices known to enhance the relevance of course material to students, potentially improving student engagement. Curricula developed with these priorities can help teachers become more effective in teaching content as they build on successful pedagogical practices and support historically underserved students' engagement with mathematics.

Districts need information sources that provide a more comprehensive assessment of curricula.

Districts planning to adopt a new curriculum need information to help them determine whether a prospective curriculum aligns with their priorities. Ideally, districts would be easily able to use a single comprehensive information source to make curriculum decisions.

Because, unlike the other tools, EdReports is publicly available, it finds widespread application in curriculum decision making. Its breadth of measurement domains and intuitive green-yellow-red coding system make it an easily accessible tool and source for curriculum information. However, our analysis reveals two areas, noted below, in which EdReports could be refined or expanded to provide more comprehensive information:

- When considering alignment to the CCSS, EdReports does not analyze every task within each lesson in the curriculum. Refining EdReports measures of CCSS alignment to include a more fine-grained analysis may provide a more nuanced differentiation that allows districts to make better-informed decisions about curriculum selection.
- EdReports does not currently assess CRMT guidance. Cultural responsiveness of instruction is a relatively new area of growing interest in the education field. Incorporating CRMT measures into EdReports, as measurement domains, may strengthen EdReports by increasing its already strong breadth, making it a more comprehensive tool that districts can rely on to capture measures related to more of their priorities.

Targeted professional learning is an important way to help teachers bridge the gaps between selected curricula and districts' strategic vision and priorities.

Given that each curriculum ranks differently across tools and domains, districts should select a curriculum featuring the strengths that align with their strategic priorities. To address areas where the curriculum may fall short, districts can and should invest in professional learning to create coherence between their vision and teachers' instructional enactment. For example, Eureka Math is a green-rated curriculum that scored well on its topic alignment with the CCSS, but it placed more emphasis on less cognitively demanding tasks and lacked instructions to support multilingual learners and differentiate instruction. A district with a large population of multilingual learners that wants to align its priority topics with the CCSS may select

Eureka Math, but it should plan to provide professional learning to support teachers in their instructional delivery aligned with district goals.

Next steps

Curriculum selection is an essential instructional consideration. Even though Illustrative Math is almost always the highest-rated curriculum, Eureka Math is better aligned with the CCSS recommendations on topic emphasis, and Into Math and California Math provide considerable guidance for engaging students' Community and Cultural Funds of Knowledge. The implication is that no single curriculum is likely to meet all of a district's needs, and the best curriculum for each district is a function of district priorities.

Below, we describe what we think interested parties should do with the above findings and implications.

- **District staff** should use the available information about curriculum characteristics to enhance their understanding of how curricula align with their vision, priorities, and goals. The available information also identifies areas in which curricula fall short such that district staff should consider supplemental materials or professional learning to help teachers implement lessons in ways that maintain a curriculum's strengths and support student engagement.
- **Teachers** should consider how to adapt their curricula to support student engagement and improve student outcomes, potentially supplemented by professional learning or supplemental materials provided by the district. Curricular adaptation could call for encouraging students to try more rigorous mathematical tasks; drawing on students' community and home knowledge, culture, or experiences; affirming positive mathematics identities for all races, genders, and ethnicities; explicitly addressing and challenging stereotypes; or providing students with mathematical tasks that involve analyzing, critiquing, or addressing meaningful issues in their lives.
- **Curriculum developers** should strengthen curricular areas that are weak relative to other areas (for example, cognitive demand and engaging student's Community and Cultural Funds of Knowledge) and that may help support teachers in engaging students in mathematics.
- **Researchers** should prioritize efforts that help the field of education understand the extent to which CRMT domains affect student outcomes.

The data analyzed in this report provide important context for instructional materials that could guide professional investments, teachers' classroom practices, and student experiences. We identify potential areas for future exploration, based on the work completed to date, as summarized below:

- **Aligning district visions with curriculum selection and professional learning.** How are district visions of high-quality mathematics instruction reflected in middle school mathematics curricula and professional learning?
- **Culturally responsive middle school mathematics curricula.** How do school and district leaders support teachers in delivering culturally responsive mathematics teaching?
- **Productive curricular adaptation.** To what extent do teachers' curricular adaptions align with mathematical learning progressions?

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Appendix A

Summaries of curricula

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Into Math

Into Math is a grade K–8 core curriculum published by HMH. Many of the print-based curriculum's components are available in digital and interactive versions. Online resources include data dashboards that help teachers monitor student learning, identify student learning needs, and guide teachers in how to differentiate instruction to meet student needs.

Into Math first emphasizes conceptual understanding and reasoning skills before connecting students' understanding to procedural practice in concepts and skills. Many lessons span several days to help students build perseverance and engage in productive struggle. The curriculum includes cross-curricular tasks throughout the grade levels to promote learning in science, technology, engineering, and mathematics (STEM).

Each lesson begins with a whole-class activity to introduce the day's task and build students' understanding through student discussion. Next, teachers lead a five- to 10-minute whole-group lesson that connects mathematical concepts and skills to the understanding developed in the initial activity. Students then engage in a quick formative assessment, allowing teachers to check student understanding. Following the assessment, students receive differentiated instruction through small groups, independent practice, or enrichment activities. All lessons end with a wrap-up activity in which the teacher brings students together to summarize what they learned.

Eureka Math

Eureka Math—also known as EngageNY—is a grade pre-K (PK)–12 core curriculum that carefully sequences mathematical progressions to present high-quality mathematics content, teaches persistence in problem solving, and prepares students to understand advanced mathematics. The curriculum emphasizes mathematical thinking and problem solving and understanding of mathematics concepts so that students can use a variety of strategies for solving difficult problems. Eureka Math provides comprehensive print and digital curriculum materials and professional development.

In 2012, Great Minds® was awarded a contract to develop a mathematics curriculum for New York State to meet the CCSS for rigor, focus, and coherence. Great Minds partnered with the New York State Education Department, mathematicians, and educators and developed EngageNY. The curriculum focuses on high-quality mathematics and the logical progression of learning from PK–12. To support teachers further, Great Minds updated and revised EngageNY to create Eureka Math. Eureka Math features the same curriculum structure and sequence as EngageNY but also provides a suite of resources to support teachers, students, and families. In addition, Eureka Math offers several resources designed for hybrid and virtual learning environments.

The entire PK–12 Eureka Math curriculum, along with a variety of instructional materials and support resources, can be downloaded at no charge. Some materials, such as printed workbooks, the Eureka digital suite, and Affirm (the formative assessment package), require a fee.

Each lesson begins with a warm-up activity that builds the needed procedural practice before the lesson connects procedure to mathematical concepts. Then, two to three 10- to 15- minute exercises connect to mathematical concepts, each of which is followed by a set of practice problems. Students then complete an exit ticket that teachers can use to check for student understanding.

Illustrative Mathematics

Illustrative Mathematics is a grade K–12 core curriculum developed by Illustrative Mathematics that is available in digital and print versions. It is a problem-based curriculum designed to help students learn by doing mathematics, solving problems in mathematical and real-world contexts, and constructing arguments by using precise language. Teachers facilitate student learning with research-based mathematical routines to guide learners in understanding and making connections between concepts and procedures.

As each unit progresses, teachers systematically introduce students to representations, contexts, concepts, language, and notation. As their learning progresses, students make connections between different representations and strategies and see and understand more efficient methods of solving problems, supporting the shift toward procedural fluency and deeper conceptual understanding. The curriculum uses distributed-practice problems for ongoing practice and to support procedural proficiency.

The curriculum is published by three publishers: Kendall Hunt, LearnZillion/Imagine Learning,[16] and McGraw Hill.

- Kendall Hunt offers a free digital curriculum for grade K–8 students as well as Algebra 1, Geometry, and Algebra 2 curricula.
- Imagine Learning (formerly LearnZillion) offers digital and print formats that include easy-to-use lesson plans, built-in teacher guidance, and student materials for grade K–8 students as well as Algebra1, Geometry, and Algebra 2 curricula.
- McGraw Hill publishes its curriculum in a print, digital, and hybrid form and offers the option to bundle with ALEKS® Personalized Learning. The curriculum is targeted to grade 6–12 students.

In addition to the three publishers, Open-Up Resources offers a free digital program for grades 6–8 that is authored by Illustrative Mathematics.

Each lesson begins with a warm-up activity to introduce the day's objective and engage students in reflective thinking. Then, students build mathematical concepts and skills through two to three 15- to 20-minute activities facilitated by the teacher. Students then engage in a quick warm-down that teachers can use to check student understanding and assign optional mathematical problems for additional practice. Throughout the teacher's guide, task narratives and lesson syntheses provide teachers with guidance to summarize what students have learned.

Big Ideas Math

Big Ideas Math is a grade K–12 core curriculum published by Big Ideas Learning. Many of the print-based curriculum components are available in digital version. The publisher promotes a blended implementation that uses both print and online resources to support differentiated and standards-aligned instruction.

Big Ideas Math emphasizes focus, coherence, and rigor. The curriculum combines direct instruction with inquiry activities to help students move from concrete to abstract understanding and to learn to apply mathematical concepts in real life. The curriculum's approach to coherence involves designing each grade's content to build on the concepts learned in the earlier grades, with minimum repetition. The

[16] We reviewed the Illustrative Math curriculum published by LearnZillion/Imagine Learning.

attention to rigor means that each lesson addresses conceptual understanding, procedural fluency, and application of mathematics to real-life problems.

Each lesson begins with a whole-class instruction whereby the teacher introduces students to the day's lesson and concept. The teacher connects the day's lesson to past learning and then poses an essential question to engage students and begins building conceptual understanding. The teacher guides the class through practice problems before students work individually, in pairs, or small groups to practice what they have learned. Each lesson ends with a closure activity that brings students back together to summarize what they learned. All lessons provide guidance to teachers in how to differentiate instruction for diverse student needs.

California Glencoe Math

California Glencoe Math (California Math) is a core curriculum for grade 6–8 published by McGraw Hill Education. It is based on Glencoe Math but has undergone revision to align specifically with California's state mathematics standards. Many of the print-based curriculum's components are available in both print and digital versions. The publisher promotes a blended implementation that uses both print and online resources.

California Math embeds conceptual understanding, application, and procedural fluency in each lesson and assessment. Each lesson provides opportunities to apply mathematical concepts in real life, with assessments administered before, during, and after lessons to help teachers determine their students' proficiency. Both print and digital curriculum materials strongly encourage differentiated instruction to support students at or below level while simultaneously challenging beyond-level students.

Each lesson begins with a launch activity to engage students and help them begin to explore the day's concept. The teacher can present the activity to the whole class, in small groups, or as, a think-pair-share or independent activity. Afterward, the teacher introduces and explains the concept to the whole class. The teacher then guides student practice on several problems, with students working in a whole-class group, small groups, pairs, or independently, depending on differentiated needs. Each lesson ends with a formative assessment activity, such as an exit ticket and additional independent practice problems. All lessons guide teachers in how to differentiate instruction for diverse student needs.

Key Elements of Mathematics Success

KEMS is a core grade 3–8 curriculum developed by National Training Network, a mathematics professional development company that provides professional development and coaching services to mathematics teachers. Lessons emphasize a variety of representations of mathematics content, structured scaffolding of problem-solving techniques, student cooperative learning, and connections among concrete, pictorial, verbal, and abstract representations of mathematics content.

Each grade 6–8 curriculum contains 12 modules tied to different Core Curriculum strands, organized around essential questions, student objectives, understandings, and common misconceptions. The 12 modules require roughly five to 15 days of instruction. Modules also contain a detailed lesson plan for each day, including a warm-up activity for students, teacher modeling, a "guided discovery" group practice, independent practice, and homework. Supplementary activities are available so that teachers can provide students with additional, extended support (such as word walls), along with online materials such as video clips of a teacher demonstrating a group activity for the lesson. A student workbook follows the lessons closely and is tied to a teaching guide that follows the same day-to-day lesson plans.

Daily lessons include independent practice of earlier skills as reinforcement, a series of guided activities (such as using manipulatives) in student pairs, and more practice, with an aligned quiz and homework. Lessons are grouped into roughly one-week segments with a single set of homework problems and related activities for the week and at least one quiz or mini-assessment per week. The curriculum also includes module-level teacher scope-and-sequence guides that present the CCSS objectives, provide example problems that students should be able to solve by the end of each lesson, demonstrate how one lesson fits into another, and summarize day-to-day lesson plans that appear in the teaching guides.

Sources for the standards and objectives, assessments, activities, and unit and lesson plans in the curriculum include the Common Core Standards Initiative, North Carolina Mathematics Wiki, Illustrative Mathematics, the Partnership for Assessment of Readiness for College and Careers, Ohio Department of Education, Utah Education Network, and Noyce Foundation.

Appendix B

Description of tools and methods

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EdReports

In the AMS study, high-quality mathematics curricula are defined as those rated green by EdReports. Educator-led review teams (typically teams of five) review the curricula to assess the extent to which materials align with Common Core State standards and the extent to which teachers and students think the materials are usable. Teams review the materials independently and then meet weekly to discuss evidence. The reviews take place in a sequential process through three phases:

- **Phase 1: Focus and Coherence.** Criteria related to focus determine whether instructional materials assess the appropriate grade-level content and devote the large share of class time to the major mathematics topics for each grade. Criteria related to coherence determine whether instructional materials are consistent with progressions of the CCSS and are coherent within a single grade. Instructional materials that meet or partially meet the expectations for Phase 1 are reviewed in Phase 2.
- **Phase 2: Rigor and Mathematical Practices.** Criteria related to rigor determine if each grade's instructional materials reflect the balances set forth in the CCSS, helping students develop conceptual understanding, procedural skill and fluency, and application. Criteria related to mathematical practice determine how well materials meaningfully connect the Standards for Mathematical Content and the Standards for Mathematical Practice. Instructional materials that meet expectations for Phases 1 and 2 are reviewed in Phase 3.
- **Phase 3: Instructional Supports and Usability.** The criteria determine how well instructional materials support student learning and engagement and support teacher learning and understanding of the standards. Other criteria include usability of assessments and incorporation of technology into the instructional materials.

Instructional materials can meet, partially meet, or not meet the expectation for each phase. Definitions follow:

- **Meets expectations.** A score of meets expectations means that the instructional materials clearly and compellingly support the given phase. The final report contains a rationale describing how the instructional materials meet expectations and is supported by evidence from the materials. In some cases where the full intent of the indicator is not met, evidence may be noted if it does not affect scoring.
- **Partially meets expectations.** A score of partially meets expectations means that some elements within the instructional materials support the given phase and some do not. In this case, the final report contains two rationales, one describing where the instructional materials meet expectations and one describing where the instructional materials do not meet expectations, both with compelling evidence.
- **Does not meet expectations.** A score of does not meet expectations means that the instructional materials do not contain clear and compelling support for the phase. The final report provides a rationale that clearly articulates why the materials do not meet the phase's criteria and is supported by evidence from the materials.
- **No rating.** Some indicators are reported through qualitative evidence but are not scored.

Three of the study curricula are green-rated by EdReports and three are not. The green-rated curricula are Into Math, Eureka Math, and Illustrative Mathematics. The non–green-rated curricula are Big Ideas,

California Math, and KEMS. In Exhibit B.1., we provide the EdReports ratings for Big Ideas and California Math; EdReports has not reviewed KEMS. These six curricula were selected for the study based on the intersection between curricula of interest to the foundation and curricula used in districts willing and able to participate in the study with district schools using at least two of the curricula.

Exhibit B.1. EdReports ratings on study curricula (grade 6–8)

		Target curricula			Comparison curricula		
Phases	Grade level	Into Math (2020)	Eureka Math (2015)	Illustrative Mathematics (2018-2019)	Big Ideas Math (2013)	California Math (2014)	KEMS
Focus and coherence	6	14/14	14/14	14/14	5/14	8/14	Not rated
	7	14/14	14/14	14/14	6/14	13/14	
	8	14/14	14/14	14/14	10/14	13/14	
Rigor and mathematical practices	6	17/18	16/18	18/18	n.a.	7/18	Not rated
	7	17/18	16/18	18/18	n.a.	7/18	
	8	17/18	16/18	18/18	11/18	8/18	
Instructional supports and usability	6	35/38	25/38	38/38	n.a.	n.a.	Not rated
	7	35/38	25/38	38/38	n.a.	n.a.	
	8	35/38	25/38	38/38	n.a.	n.a.	
Alignment rating	6	Meets	Meets	Meets	DNM	DNM	Not rated
	7	Meets	Meets	Meets	DNM	DNM	
	8	Meets	Meets	Meets	Partially	Partially	

Note: Each grade is reported separately for each curriculum; grade 6 is the first set of numbers in each row, and grade 8 is the last set of numbers in each row. Color coding matches EdReports ratings. Alignment ratings are Meets Expectations (Meets), Partially Meets Expectations (Partially), or Does Not Meet Expectations (DNM). EdReports has not rated KEMS. In the report, we translate numerical ratios to percentages by summing each grade band's score and dividing by the sum of total possible points across grade bands.

n.a. = not applicable.

Surveys of Enacted Curriculum

To understand whether, how, and what features of the study curricula support or challenge high quality instructional delivery, a team of mathematics education experts with the Center for Curriculum Analysis (CCA) scored the study curricula using the Surveys of Enacted Curriculum analysis tool. The tool assesses each curriculum's alignment to the CCSS content and mathematical practice expectations and issues a "Coarse Grain" summary measure we refer to as overall alignment.

Within the coarse-grain summary, the tool assesses the extent to which the mathematical content topics emphasized in the CCSS—such as Probability, Basic Algebra, and Operations–are similarly emphasized in a curriculum. In Exhibit B.2, we show the full list of topics and their components that are used to evaluate topic emphasis.

Exhibit B.2. Topics and subtopics in SEC

Number sense, properties, and relationships	Operations	Measurement	Basic algebra
Place value	Add, subtract whole numbers, integers	Use of measuring instruments	Absolute value

Whole numbers, integers	Multiplication whole numbers, integers	Theory (arbitrary, standard units, unit size)	Use of variables
Operations	Division whole numbers, integers	Conversions	Evaluation of formulas, expressions, equations
Fractions	Combinations of add, subtract, multiply, divide by whole numbers or integers	Metric (SI) system	One-step equations
Decimals	Equivalent and non-equivalent fractions	Length, perimeter	Coordinate plane
Percentages	Add, subtract fractions	Area, volume	Patterns
Ratio, proportion	Multiply fractions	Surface area	Multistep equations
Patterns	Divide fractions	Direction, location, navigation	Inequalities
Real and rational numbers	Combinations of add, subtract, multiply, divide fractions	Angles	Linear, nonlinear relations
Exponents, scientific notation	Ratio, proportion	Circles (for example, pi, radius, area)	Rate of change, slope, line
Factors, multiples, divisibility	Representations of fractions	Mass (weight)	Operations on polynomials
Odds, evens, primes, composites, square numbers	Equivalence of decimals, fractions, percentages	Time, temperature	Factoring
Estimation	Add, subtract decimals	Money	Square roots and radicals
Number comparisons	Multiply decimals	Derived measures (for example, rate and speed)	Operations on radicals
Order of operations	Divide decimals	Calendar	Rational expressions
Computational algorithms	Combinations of add, subtract, multiply, divide decimals	Accuracy, precision	Multiple representations
Relationships between operations	Computing with percents	Volume	Coordinate plane graphs
Number theory (for example, base-ten, non–base-ten systems)	Computing with exponents, radicals	Distance	Writing expressions and equations
Mathematical properties (for example, distributive property)	Writing expressions and equations		

Probability	Advanced geometry	Analysis	Trigonometry	Special topics
Simple probability	Logic, reasoning, proof	Sequences and series	Basic ratios	Sets
Compound probability	Loci	Limits	Radian measure	Logic
Conditional probability	Spheres, cones, cylinders	Continuity	Right triangle trigonometry	Mathematical induction
Empirical probability	Coordinate geometry	Rates of change	Law of sines, cosines	Linear programming
Sampling, sample spaces	Vectors	Maxima, minima, range	Identities	Networks
Independent and dependent events	Analytic geometry	Differentiation	Trigonometric equations	Iteration, recursion
Expected value	Non-Euclidean geometry	Integration	Polar coordinates	Permutations, combinations
Binomial distribution	Topology	Kinematics	Periodicity	Simulations
Normal curve	Geometric properties		Amplitude	Fractals
Randomness	Geometric constructions			Problem-solving strategies

Advanced algebra	Statistics	Geometric concepts	Data displays	Functions
Quadratic equations	Mean, median, mode	Basic terminology	Summarize data in a table or graph	Notation
Systems of equations	Variability, standard deviation, range	Points, lines, rays, segments, vectors	Bar graph, histogram	Relations
Systems of inequalities	Line of best fit	Patterns	Pie charts, circle graphs	Linear
Compound inequalities	Quartiles, percentiles	Congruence	Pictographs	Quadratic
Matrices, determinants	Bivariate distribution	Similarity	Line graphs	Polynomial
Conic sections	Confidence intervals	Parallels	Stem-and-leaf plots	Rational
Rational, negative exponents, radicals	Correlation	Triangles	Scatter plots	Logarithmic
Rules for exponents	Hypothesis testing	Quadrilaterals	Box plots	Exponential
Complex numbers	Chi square	Circles	Line plots	Trigonometric, circular
Binomial theorem	Data transformation	Angles	Classification, Venn diagrams	Inverse
Factor and remainder theorem	Central limit theorem	Polygons	Tree diagrams	Composition
Field properties of real number system	Sample size	Polyhedra	Tally charts	Definition
Multiple representations	Statistical questions (for example, validity and reliability)	Models	Frequency table	Piece-wise functions
Logarithmic properties		3D relationships		Transformations
Rational equations		Symmetry		
		Transformations (for example, flips, turns)		
		Pythagorean theorem		

In addition, for the dimension of cognitive demand, the SEC calculates a curriculum's level of alignment to the CCSS mathematical practice expectations. Using five categories, it categorizes the cognitive demand of student performance expectations specified in a curriculum: (1) memorize or recall; (2) perform procedures; (3) demonstrate understanding; (4) conjecture, generalize, or prove; and (5) solve nonroutine problems or make connections. In Exhibit B.3, we show the full list of cognitive demand categories and examples of tasks used to evaluate cognitive engagement.

Exhibit B.3. Cognitive demand categories for mathematics

Memorize facts, definitions, formulas	Perform procedures	Demonstrate understanding of mathematical ideas	Conjecture, analyze, generalize, prove	Solve nonroutine problems/make connections
Recite basic mathematical facts	Use numbers to count, order, denote	Communicate mathematical ideas	Determine the truth of a mathematical pattern or proposition	Apply and adapt a variety of appropriate strategies to solve nonroutine problems
Recall mathematics terms and definitions	Do computational procedures or algorithms	Use representations to model mathematical ideas	Write formal or informal proofs	Apply mathematics in contexts outside mathematics
Recall formulas and computational procedures	Follow procedures/instructions	Explain findings and results from data analysis strategies	Recognize, generate, or create patterns	Apply to real-world situations
	Solve equations/ formulas/routine word problems	Develop/explain relationships between concepts	Find a mathematical rule to generate a pattern or number sequence	Synthesize content and ideas from several sources
	Organize or display data	Show or explain relationships between models, diagrams, and/or other representations	Make and investigate mathematical conjectures	
	Read or produce graphs and tables		Identify faulty arguments or misrepresentations of data	
	Execute geometric constructions		Reason inductively or deductively	

CRMT curriculum coding tool

To understand whether and how the study curricula facilitate culturally responsive instruction and create opportunities for students to demonstrate their learning in holistic ways, we used an adaptation of the Culturally Responsive Mathematics Teaching (CRMT) 2 Lesson Analysis Tool (Aguirre and del Rosario Zavala, forthcoming) that we call the CRMT curriculum coding tool. The CRMT curriculum coding tool was designed to support professional development during classroom observation and subsequent teacher meetings, but we adapted it as a means of quantitatively scoring instructional materials.

We used the CRMT curriculum coding tool to assess curricula across nine domains that measure three broad categories: knowledge and identities; rigor and support; and power and participation. Each domain is measured on a rubric scale (1–5) that defines the type of curriculum evidence necessary, within a

lesson, to meet thresholds for opportunities to enact CRMT. Each rubric includes an overarching question that captures essential elements of the domain (Exhibit B.3).

Exhibit B.3. Culturally responsive measurement categories, domains, and their essential questions

Categories	Domains	Essential question
Knowledge and identities	Community and Cultural Funds of Knowledge (CFoK)	How does the lesson as written help students connect mathematics with meaningful issues or situations in their lives?
	(Re) Humanizing	How does the lesson as written support creativity, broaden what counts as mathematical knowledge, and affirm positive mathematics identities for all students?
	Student Ideas and Thinking	How does the lesson as written create opportunities to elicit, express, and build on student mathematical thinking in a variety of ways?
Rigor and support	Cognitive Demand	How does the lesson as written enable all students to closely explore and analyze mathematics concept(s), procedure(s), and problem-solving or reasoning strategies?
	Maintaining Rigor	How does the lesson as written maintain high rigor with strong support for all students?
	Affirming Multilingualism	How does the lesson as written position multilingual learners (MLL) as competent learners in mathematics activities?
Power and participation	Distributing Intellectual Authority	How does the lesson as written distribute mathematics authority and make space for a variety of forms of knowledge and communication?
	Disrupting Power	How does the lesson as written disrupt status differences, entrenched stereotypes, and inequitable power relationships present in all mathematics classrooms?
	Taking Action	How does the lesson as written support students' use of mathematics to analyze, critique, and address power relationships and injustice in their lives (economic, social, environmental, legal, political, patriarchal)?

We developed the scale for each domain by reviewing each curriculum and comparing expected evidence for each item to the actual evidence within each curriculum. Within the nine domains, we defined evidence for how a curriculum can provide guidance, support, or opportunities for teachers to implement culturally responsive strategies and pedagogies (Exhibit B.4.).

Exhibit B.4. Curriculum evidence for CRMT domains

Domains	Curricula evidence. . .
Community and Cultural and Funds of Knowledge (CFoK)	• Reference students' community and home knowledge, culture, or experiences • Inquire about student backgrounds and experiences • Ask students to reflect on instances in which they might have seen a topic in their own life, ask them to discuss the experience, and adapt it as a problem for the class
(Re) Humanizing	• Affirm positive mathematics identities for all races, genders, and ethnicities • Include mathematical problems that honor students' diversity and culture or include how other cultures or communities have used mathematics to honor their traditions • Acknowledge that individuals around the world have been successfully involved with mathematics in various ways for centuries

Domains	Curricula evidence. . .
Student Ideas and Thinking	• Include guidance for teachers to encourage students to share their reasoning, ask questions of one another, discuss each other's ideas, or promote a shared understanding across the whole class • Include guidance for teachers to encourage and value a variety of forms of communication, including hand gestures, pictures or drawings, and diverse responses
Cognitive Demand	• Include mathematical tasks that emphasize underlying concepts, patterns, and properties • Include tasks that require students to explain their reasoning
Maintaining Rigor	• Ensure that scaffolds do not lower the expectation for some students • Include opportunities for students assigned lower-level tasks to reconnect to rigorous content
Affirming Multilingualism	• Include prompts for teachers to encourage students to lean on their linguistic resources and home language to help make sense of mathematics • Include guidance for teachers to lift up the language that students use and share it with the class • Reference strategies that develop academic language, such as repeating all or part of what a student said to ensure understanding or using graphic organizers to visualize or present information in a way that is easy to comprehend
Distributing Intellectual Authority	• Include prompts for teachers to encourage students to draw on their experiences and knowledge to make sense of and connect with the mathematics concepts they are learning • Ensure that mathematical tasks include opportunities for students to share their reasoning with one another, whether in pairs or small groups • Include guidance for teachers to encourage student input and ownership through inquiry-based instruction
Disrupting Power	• Include teacher prompts for explicitly addressing and challenging stereotypes • Include teacher guidance for using inclusive talk that builds up students instead of tearing down ideas or insisting on one correct way • Include teacher prompts for implementing classroom norms to ensure that each student speaks during a lesson
Taking Action	• Provide students with mathematical tasks that involve analyzing, critiquing, or addressing an issue they strongly connect to, such as a topic on social justice • Provide students with mathematical problems related to current or historical issues of injustice or social justice, such as calculating the cost of buying bottled water when lead is discovered in the water system or the cost of rent when families are displaced by hurricane damage to their homes

Within each curriculum's grade 6 materials, we selected four units that covered the following topics: number sense, operations, measurement, and data displays. Within each unit, we coded nine 50-minute lessons[17]—three from the beginning of each unit, three from the middle, and the last three lessons of the unit (excluding test days). We coded 36 lessons across four units in each of the study curricula,[18] for which two to three coders independently reviewed the teacher's guide and noted evidence or opportunities to enact CRMT aligned with each domain.[19] After coding, the coders came to consensus on any misaligned codes before finalizing the data.

[17] With KEMS designed for 80- to 90-minute lessons, we coded six lessons in order to code an amount of content equivalent to that of the other curricula, two from the beginning, middle, and end.

[18] The study team coded a total of 203 lessons.

[19] We did not review other supplementary material (such as a guide for students with disabilities or English language learners).

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Appendix C

Results by tool

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In this appendix, we provide supplemental analyses that give more detailed information from the SEC and CRMT curriculum analyses. In Exhibit C.1, we display each curriculum's scores across each measurement tool.

Exhibit C.1. Curriculum measure results

Curriculum	EdReports[1]				SEC[2]			CRMT curricula coding domains[3]								
	Focus and Coherence	Rigor and Math Practices	Alignment	Usability	Overall Alignment	Topic emphasis	Cognitive Demand	1 CFoK	2 (Re) Human	3 Stu Think	4 Cog Dem	5 Rigor	6 Affr. ML	7 Intell. Auth.	8 Power	9 Taking Action
Illustrative Math	14/14	18/18	Meets	38/38	69%	2.93%	28%	1.1	1.0	4.3	3.9	4.7	3.0	4.3	1.0	1.0
	14/14	18/18	Meets	38/38												
	14/14	18/18	Meets	38/38												
Into Math	14/14	17/18	Meets	35/38	60%	1.89%	13%	1.3	1.0	3.0	3.2	3.6	2.5	2.9	1.0	1.0
	14/14	17/18	Meets	35/38												
	14/14	17/18	Meets	35/38												
Eureka Math	14/14	16/18	Meets	25/38	67%	1.83%	20%	1.1	1.0	3.4	3.5	1.8	1.2	3.1	1.0	1.0
	14/14	16/18	Meets	25/38												
	14/14	16/18	Meets	25/38												
Big Ideas	5/14	N/A	DNM	N/A	62%	1.89%	17%	1.1	1.0	2.9	3.0	2.8	1.9	2.6	1.0	1.0
	6/14	N/A	DNM													
	10/14	11/18	Partial													
California Math	8/14	7/18	DNM	N/A	65%	2.81%	21%	1.1	1.0	2.5	3.3	3.1	2.2	2.8	1.0	1.0
	13/14	7/18	DNM													
	13/14	8/18	Partial													
KEMS	Not Rated				60%	2.74%	21%	1.0	1.0	2.7	3.3	1.1	1.0	2.4	1.0	1.0

[1] For curriculum scores that differ by grade, grade 6 represents the first set of numbers in each row and grade 8 the last set of numbers in each row. Color coding matches EdReports ratings. Alignment ratings are Meets Expectations (Meets), Partially Meets Expectations (Partially), or Does Not Meet Expectations (DNM). Big Ideas and California Math are not rated on the Usability domain because they do not meet expectations for the first two sets of ratings.

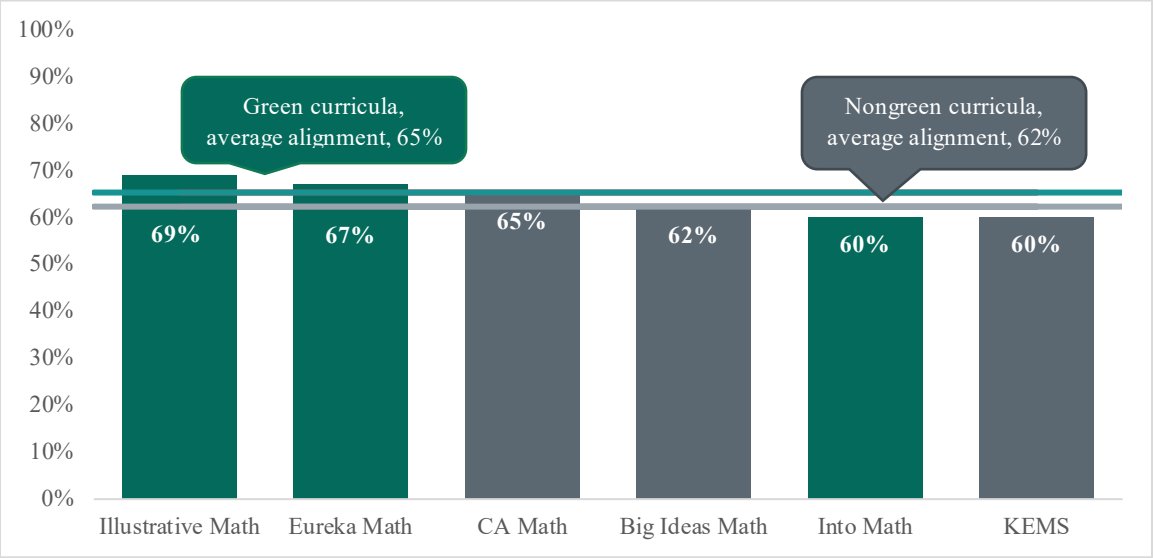
[2] Overall alignment is measured as the percentage of the curriculum that aligns with the CCSS. Topic emphasis is measured as the average absolute value difference between the content covered by the curriculum compared to the CCSS-recommended coverage. Cognitive demand percentages represent the percentage of cognitively demanding tasks included in the curriculum.

[3] Each domain score represents the average score—from 1 to 5—in each domain across the six or nine lessons coded.

Findings from SEC ratings

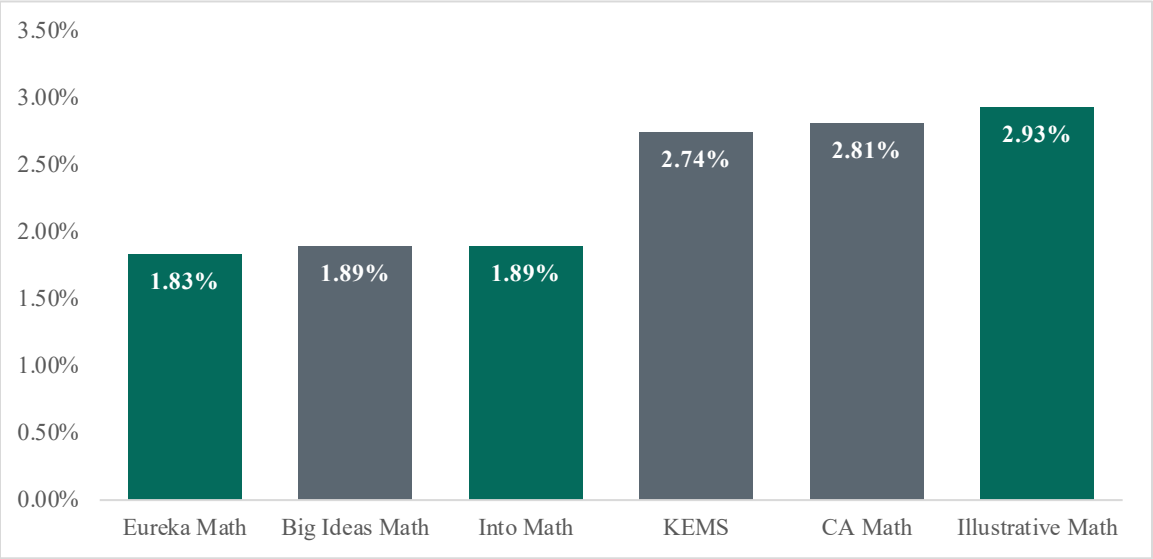
In Exhibit C.2, we show each curriculum's overall alignment with the CCSS content and mathematical practice expectations. Illustrative Math (69 percent) and Eureka Math (67 percent) had the highest overall alignment with the CCSS while KEMS (60 percent) had the lowest overall alignment.

Exhibit C.2. Overall alignment with CCSS according to SEC, by curricula



In Exhibit C.3, we present the average difference in the SEC topic emphasis from that recommended by the CCSS. We display the absolute difference of percentage emphasis that each curriculum placed on each topic area by summarizing the difference across topics. In Exhibit C.4, in contrast, we show the difference by topic. Illustrative Math had the greatest average absolute difference, meaning that it had 2.93 percent more or less topic coverage compared to the CCSS recommendations.

Exhibit C.3. Average absolute difference in SEC topic emphasis from CCSS



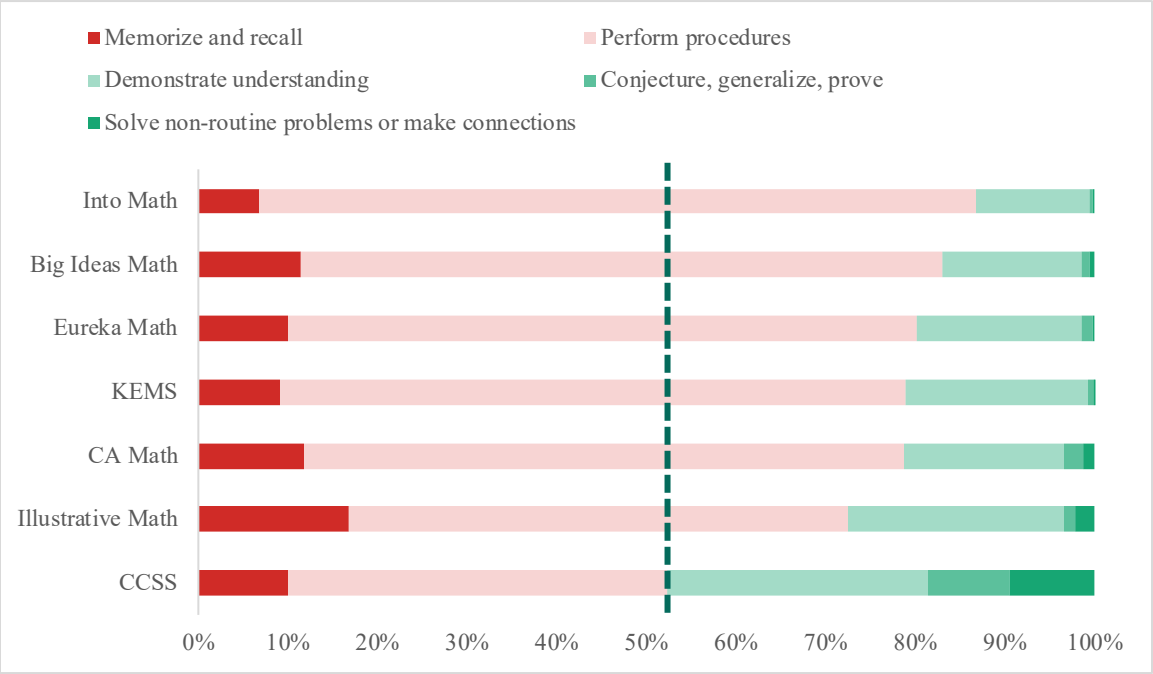
In Exhibit C.4, we display how curricula emphasize topics as compared to the CCSS. Upward arrows indicate that a curriculum places greater emphasis on a topic than recommended by the CCSS, by more than 1 percent. Downward arrows indicate that a curriculum places less emphasis on a topic than recommended by the CCSS, by more than 1 percent. An equals sign indicates when a curriculum's topic emphasis is within 1 percent of the emphasis recommended by the CCSS.

Exhibit C.4. Emphasis of each curriculum on each SEC topic area relative to CCSS recommendations for emphasis, SEC

	Green Curricula			Non-green Curricula		
SEC topic area	Illustrative Math	Eureka Math	Into Math	Big Ideas	KEMS	CA Math
Number Sense	↓	↑	↓	↑	↑	↑
Operations	↑	=	↑	↑	↑	↑
Measurement	↑	↑	↑	=	↑	=
Basic Algebra	↓	↓	↓	=	↓	↓
Advanced Algebra	↓	↓	=	=	↓	↓
Geometric Concepts	↑	↑	=	=	↓	↓
Advanced Geometry	=	=	=	=	=	=
Data Displays	↑	↑	↑	↑	↑	↑
Statistics	↓	↓	↓	↓	↓	↓
Probability	↓	↓	↓	↓	↓	↓
Functions	↓	=	↓	=	=	=

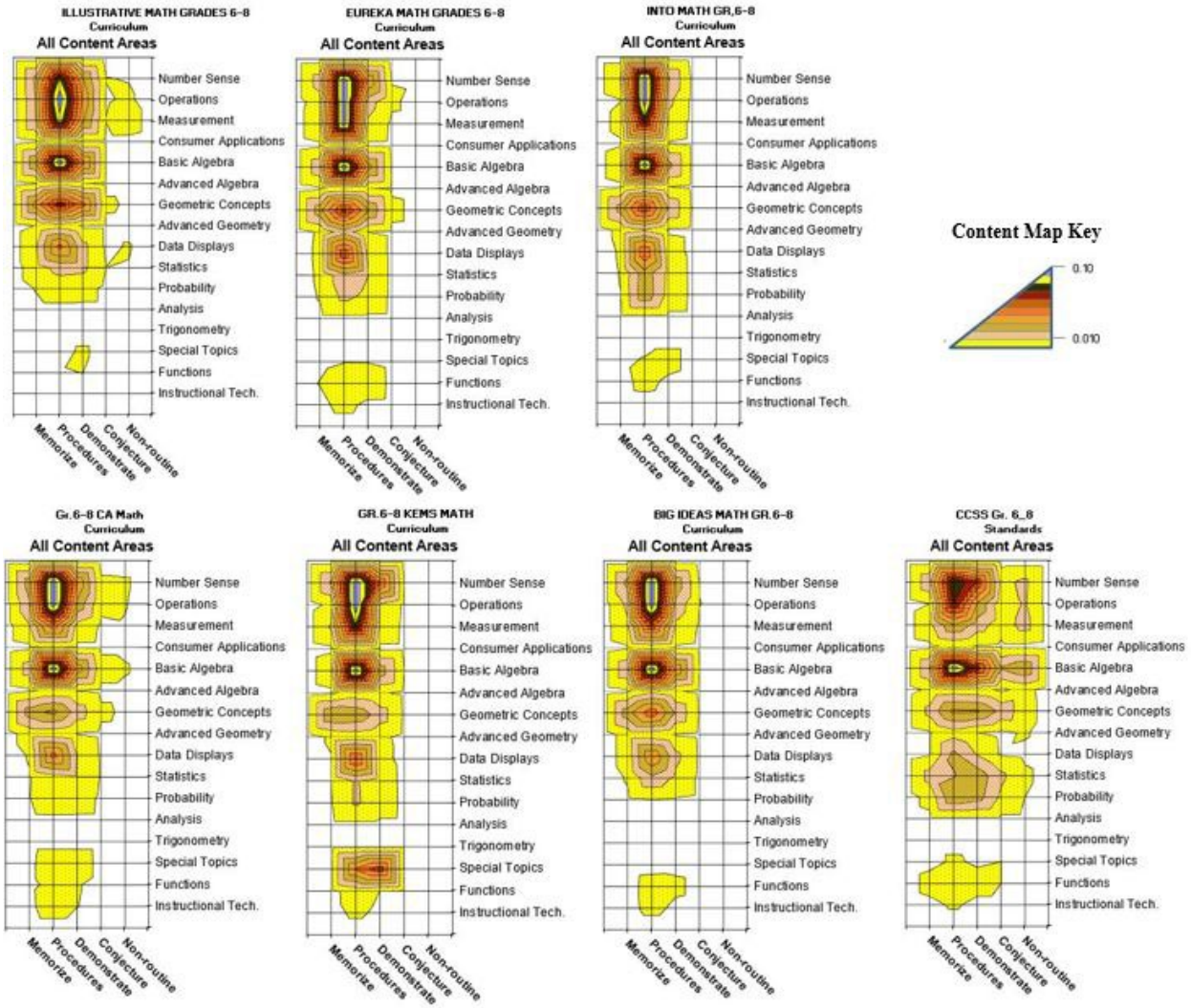
In Exhibit C.5, we present the percentage of curricula aligned with each of the five cognitive demand categories coded through the SEC. The most cognitively demanding curriculum (Illustrative Math) is almost half as cognitively demanding as recommended by the CCSS mathematical practice expectations. The area to the right of the dashed line represents the percentage of material that should include higher cognitive demand tasks (48 percent), with bars shaded in variations of red.

Exhibit C.5. Cognitive demand category emphasis by curricula



In Exhibit C.6, we show the content maps produced by the SEC for each curriculum and the CCSS. The maps show a disaggregated topic emphasis by cognitive demand category. The colors on the topographical map represent the percentage of curricula covered by each content topic within each cognitive demand category.

Exhibit C.6. Content maps of curricula



Findings from CRMT ratings

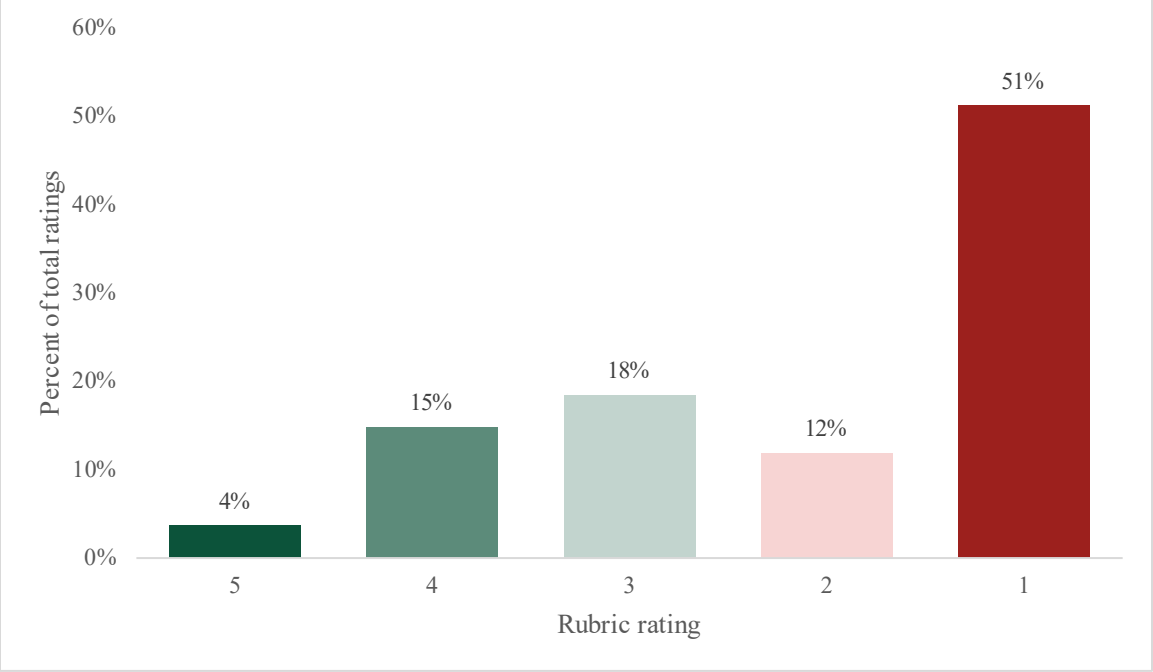
The CRMT curriculum coding tool measures the prevalence of guidance in the curriculum that connects to student knowledge and identities, provides adequate rigor and supports for students, and elevates student power and participation. In Exhibit C.7, we display the average score for each CRMT domain by curriculum, with 36 lessons rated for each curriculum.

Exhibit C.7. CRMT domain scores by curriculum

	CFoK	(Re) Human	Student Ideas	Cognitive Demand	Maintain Rigor	Affirm Multiling.	Distribute Authority	Disrupt Power	Taking Action
Illustrative Math	1.1	1.0	4.3	3.9	4.7	3.0	4.3	1.0	1.0
Into Math	1.3	1.0	3.0	3.2	3.6	2.5	2.9	1.0	1.0
Eureka Math	1.1	1.0	3.4	3.5	1.8	1.2	3.1	1.0	1.0
Big Ideas	1.1	1.0	2.9	3.0	2.8	1.9	2.6	1.0	1.0
California Math	1.1	1.0	2.5	3.3	3.1	2.2	2.8	1.0	1.0
KEMS	1.0	1.0	2.7	3.3	1.1	1.0	2.4	1.0	1.0

Each lesson received nine scores, one for each domain. Across the study curricula, we issued 1,827 ratings (we coded 203 lessons across each of nine domains). In Exhibit C.8, we present the percentage of CRMT rubric ratings for each score (1–5). For example, 51 percent of lesson ratings were a 1, representing no instructions or guidance at all pertaining to the relevant domain; 18 percent were a 3, representing one meaningful instruction that includes all students for the relevant domain.

Exhibit C.8. Percentage of CRMT rubric ratings



Appendix D

Results by curriculum

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This appendix includes one-page curriculum snapshots (Exhibits D.1.–D.6.) that provide insights into curriculum ratings across all three measurement tools as follows:

EdReports

- **Focus and Coherence.** Measures the extent to which instructional materials assess and cover the appropriate grade-level content and are consistent with progressions of the CCSS and are coherent within a single grade.
- **Rigor and Mathematical Practices.** Measures the extent to which instructional materials develop conceptual understanding, procedural skill, and fluency and apply and meaningfully connect the Standards for Mathematical Content and the Standards for Mathematical Practice.
- **Usability.** Measures the extent to which instructional materials support student learning and engagement, teacher learning and understanding, usability of assessments, and incorporation of technology into the instructional materials.

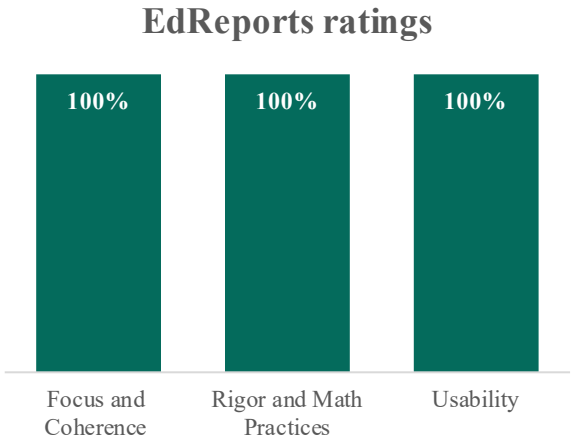
SEC

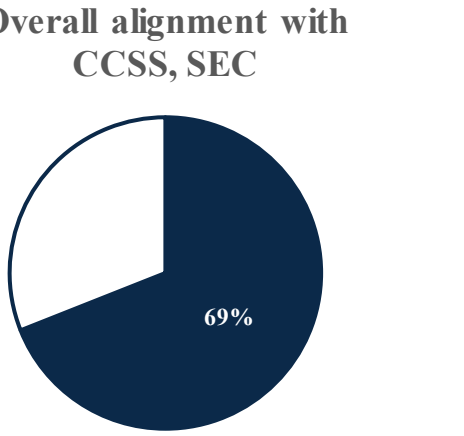
- **Overall alignment.** A curriculum's average alignment with both the CCSS content and mathematical practice expectations.
- **Higher cognitive demand.** A curriculum's average alignment with the CCSS content expectations.
- **Content coverage.** A curriculum's average alignment with the CCSS mathematical practice expectations.

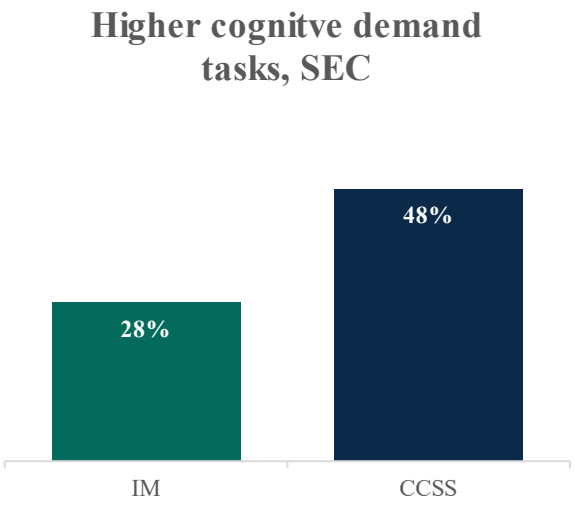
CRMT Scores

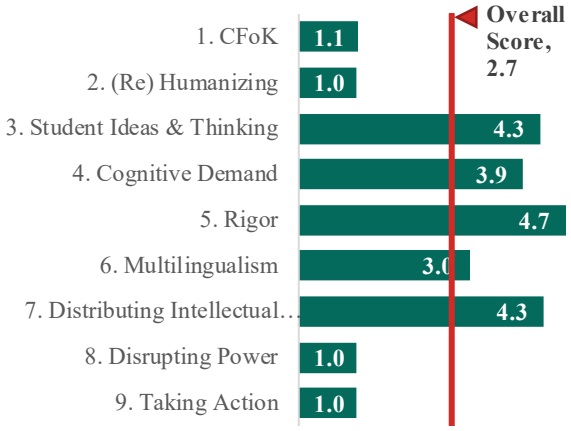
- **CRMT Scores.** A curriculum's average lesson score for each CRMT domain.

Exhibit D.1. Illustrative Math (Imagine Learning, LearnZillion, 2018–2019)









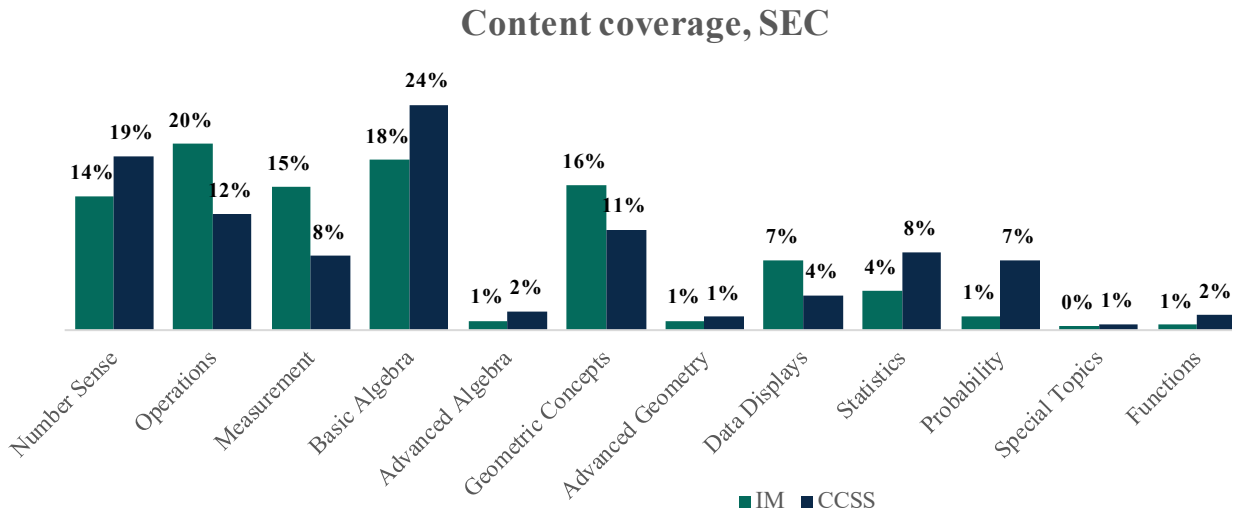
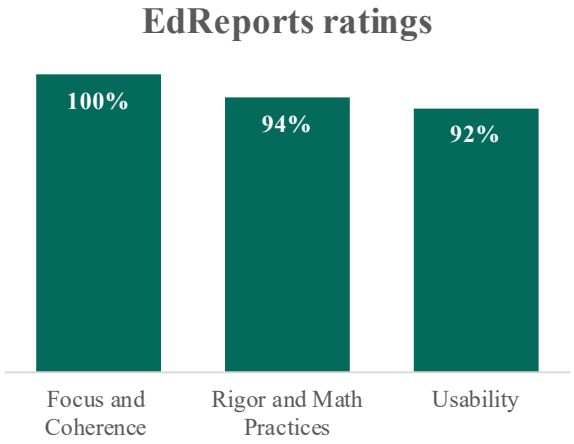
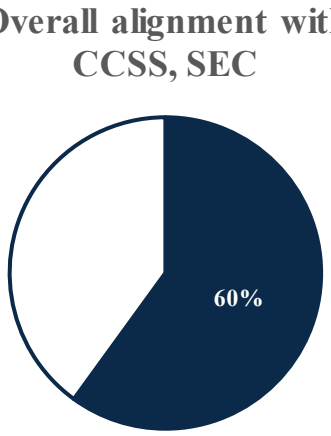
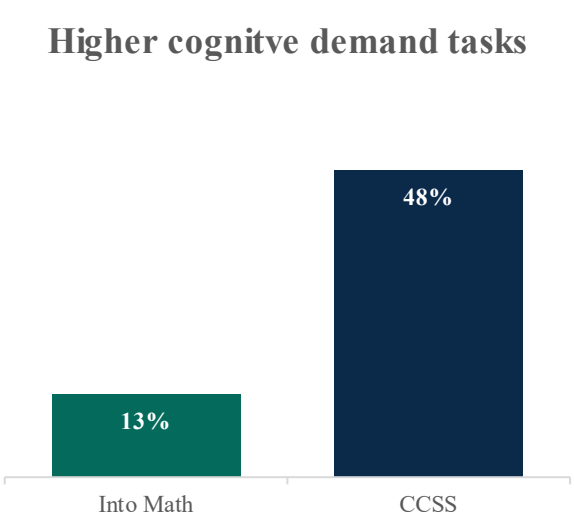
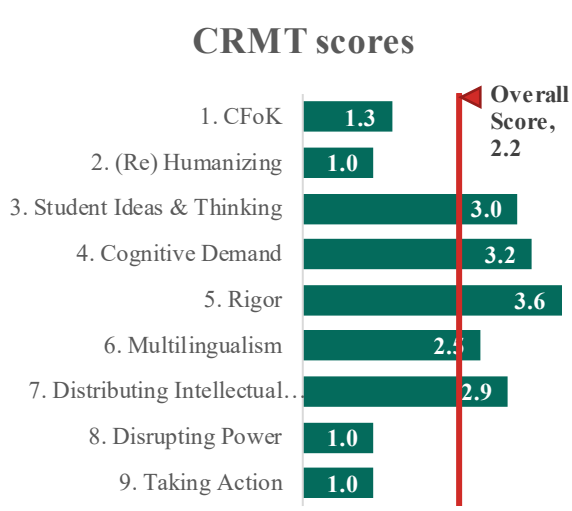


Exhibit D.2. Into Math (Houghton Mifflin Harcourt, 2020)









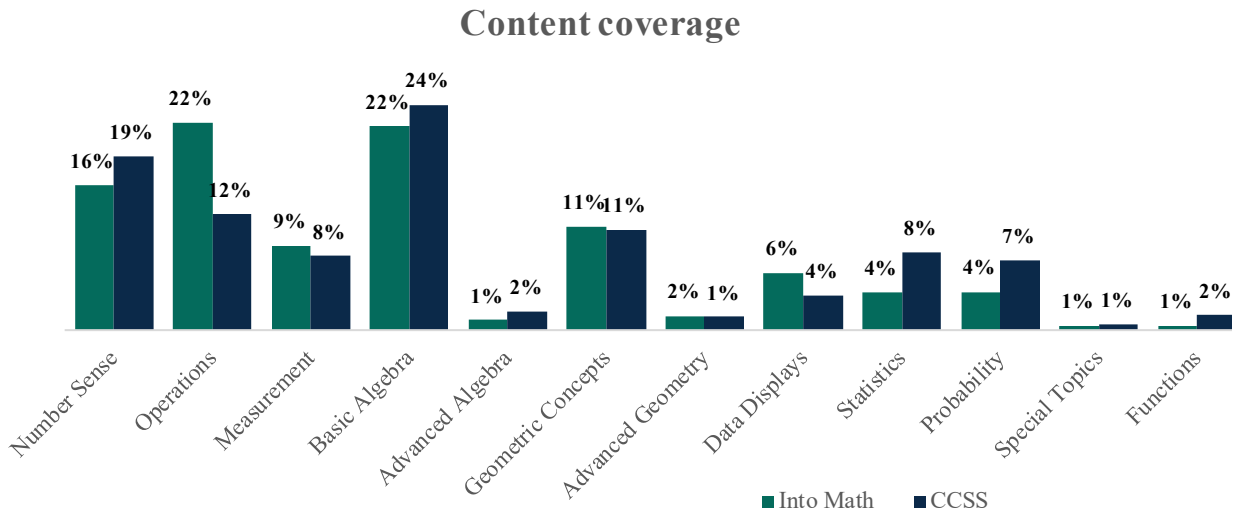


Exhibit D.3. Eureka Math (Great Minds, 2021)

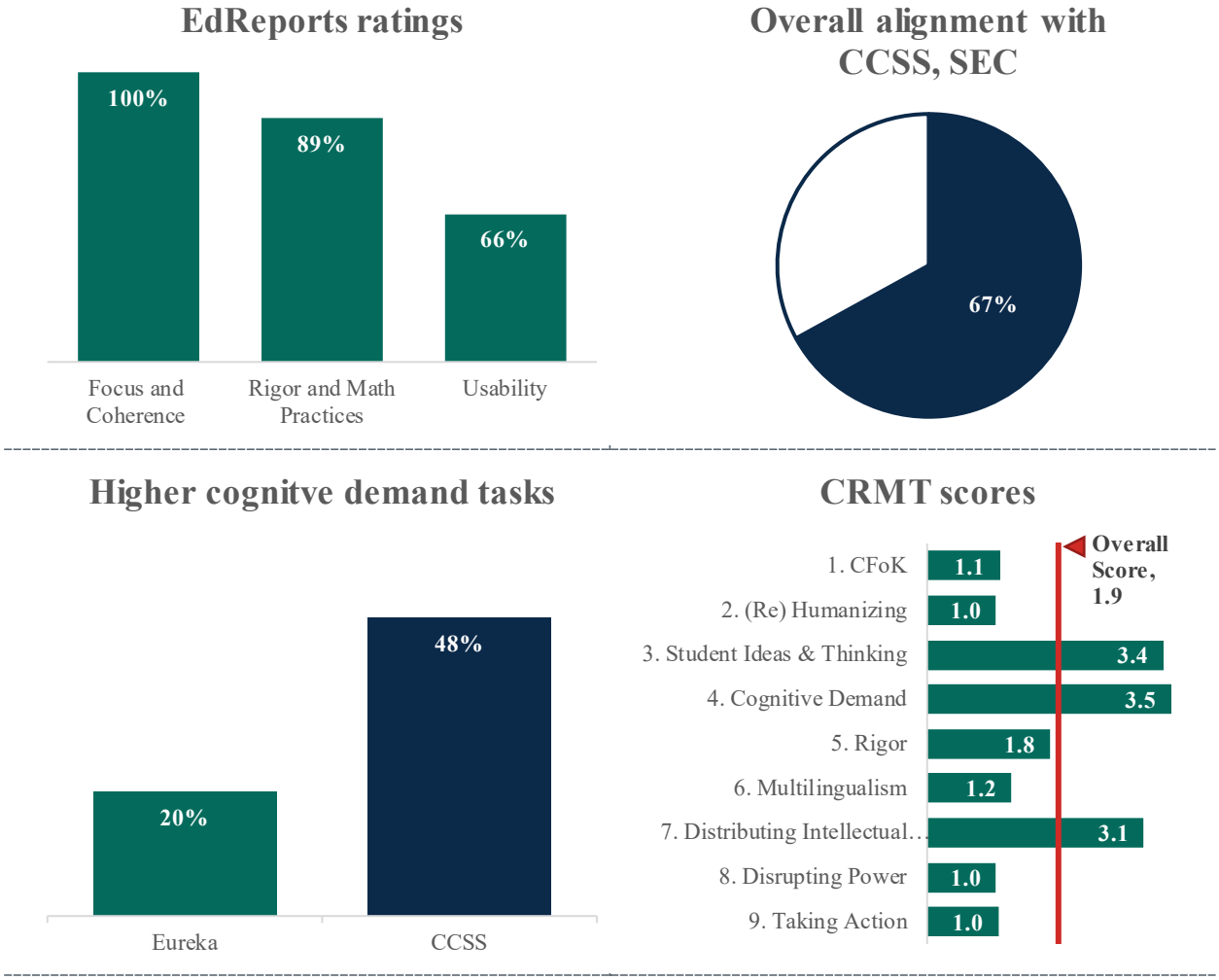
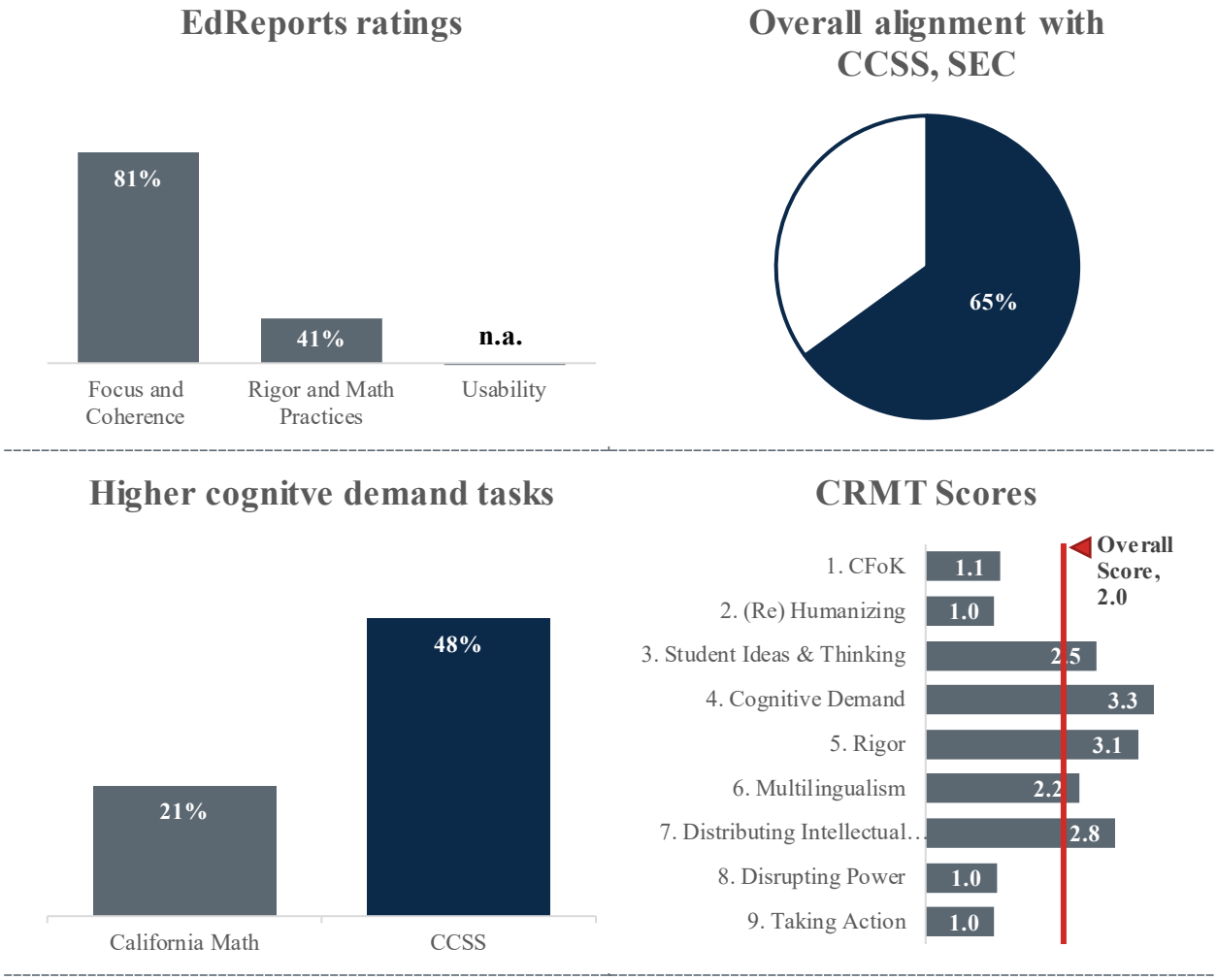
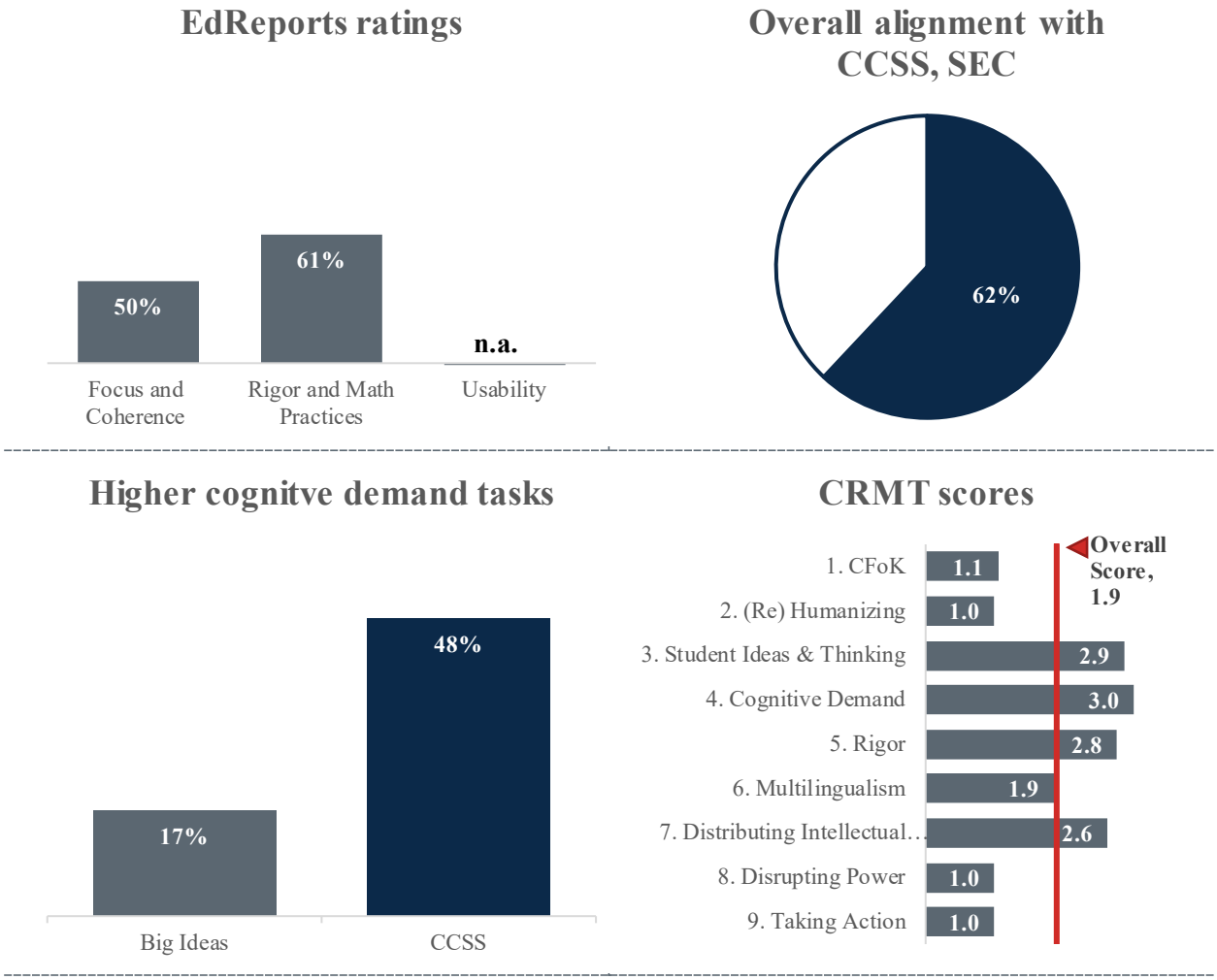


Exhibit D.4. California Math (Glencoe, McGraw-Hill Education, 2014)



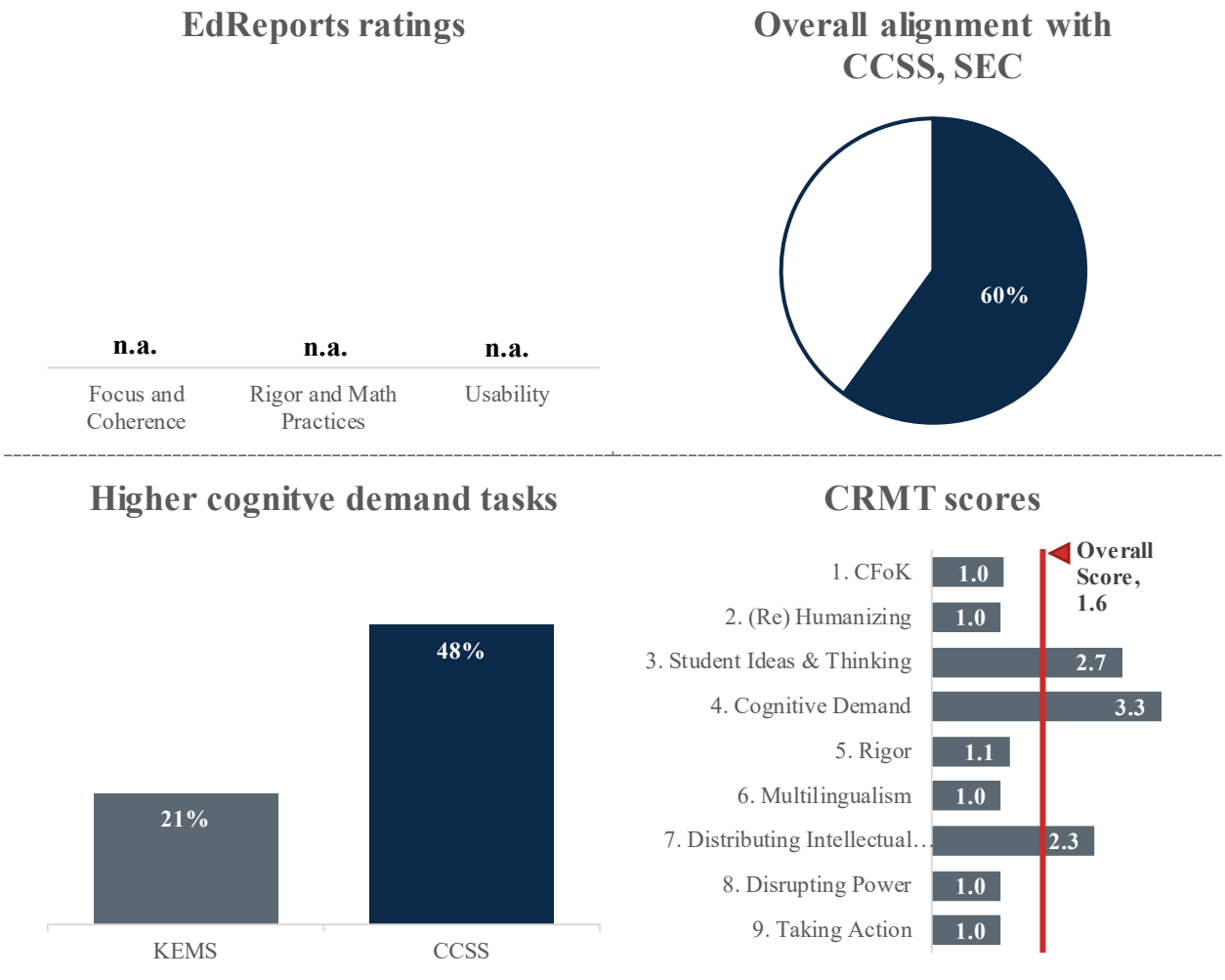
n.a. = not applicable

Exhibit D.5. Big Ideas Math (Big Ideas Learning, LLC, 2013)



n.a. = not applicable

Exhibit D.6. Key Elements of Mathematical Success (KEMS) (National Training Network, 2020)



n.a. = not applicable

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