Mathematical Learning Progressions

A study of teachers' content adaptations and alignment with mathematical learning progressions

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Abstract

This study examines whether teachers' content adaptations to middle school math curricula align with mathematical learning progressions and explores their motivations for adaptation. This study originates from a concern that as teachers make adaptations to their lessons, they may make adaptations that move students away from the intended learning goal. Using a mixed-methods approach, we found that teachers were more likely to make productive over nonproductive adaptations. Teachers who made productive adaptations were more likely to consider learning progressions and adapt lessons to below-grade-level content. Teachers who made nonproductive adaptations were more likely to adapt lessons to on-grade-level content while delivering the lesson. Our findings suggest that districts should consider curricula and professional learning that intentionally highlight mathematical learning progressions.

When Teachers Deviate from Curricula, Do They Make Productive Adaptations?

The Analysis of Middle School Math Systems (AMS) project is a mixed-methods study that explores the enabling and disabling conditions under which teachers adopt, adapt, and enact middle school mathematics curricula. The AMS project is part of a larger set of investments, funded by the Bill & Melinda Gates Foundation, intended to help students who are Black, Latino, multilingual learners, and those experiencing poverty succeed in mathematics. Situated within the AMS project, this paper explores the extent to which teachers' content adaptations[1] align with, or deviate from, mathematical learning progressions, as well as how and why teachers adapt and enact middle school mathematics curricula.

The curriculum teachers use heavily influences the content students experience and learn (Stein, 2007). Currently, the education field invests billions of dollars annually in instructional materials (NCES, 2021), which reflects the crucial influence these materials have on student learning (Squires, 2012; Parrish & Bryd, 2022; Whitehurst, 2009; Agodini et al., 2013; Agodini & Harris, 2010). Because previous research has demonstrated that mathematical curricula must be more focused and coherent (Common Core State Standards Initiative, n.d.), there has been a push to develop and publish curricula that are aligned with the Common Core State Standards for Mathematics (CCSSM; EdReports, 2023), especially because standards alignment is often used as a signal of high-quality materials (EdReports, 2023).

Even when teachers have access to standards-aligned curricula, there is little evidence that they use curricula as intended (Confrey, 2019). Moreover, teachers face a constant dilemma between enacting grade-level expectations that are embedded in curriculum standards and meeting students' learning needs when some or all of their students may not be ready for grade-level expectations (Confrey, 2019). To meet their students' needs, teachers often make adaptations to the content of the lessons. These adaptations, however, may not take into consideration the mathematical learning progressions that underpin the standards on which curricula are constructed. A concern is that teachers will move students away from the intended mathematical goal when making these adaptations. Thus, we need to investigate how teachers use, adapt, and deliver curriculum to students to better understand the extent to which the adaptations teachers make follow mathematical learning progressions.

Literature Review

Mathematical learning progressions

The idea of learning progressions that focus on children's development of content knowledge over time parallels other constructs, such as spiraling curriculum, developmental corridors, and cognitively guided instruction (Duncan, 2009). It builds on Carol et al.'s (2006) definition that "learning progressions [are] descriptions of successively more sophisticated ways of reasoning within a content domain based on research syntheses and conceptual analyses." Battista (2010) connected the idea to student learning, identifying that a student's mathematical sensemaking ultimately depends on instruction grounded in

[1] This sub-study focuses on content adaptations—as measured by the Common Core State Standards for Mathematics—rather than adaptations serving other purposes, such as adapting lessons to be more culturally responsive or translating content for English learners.

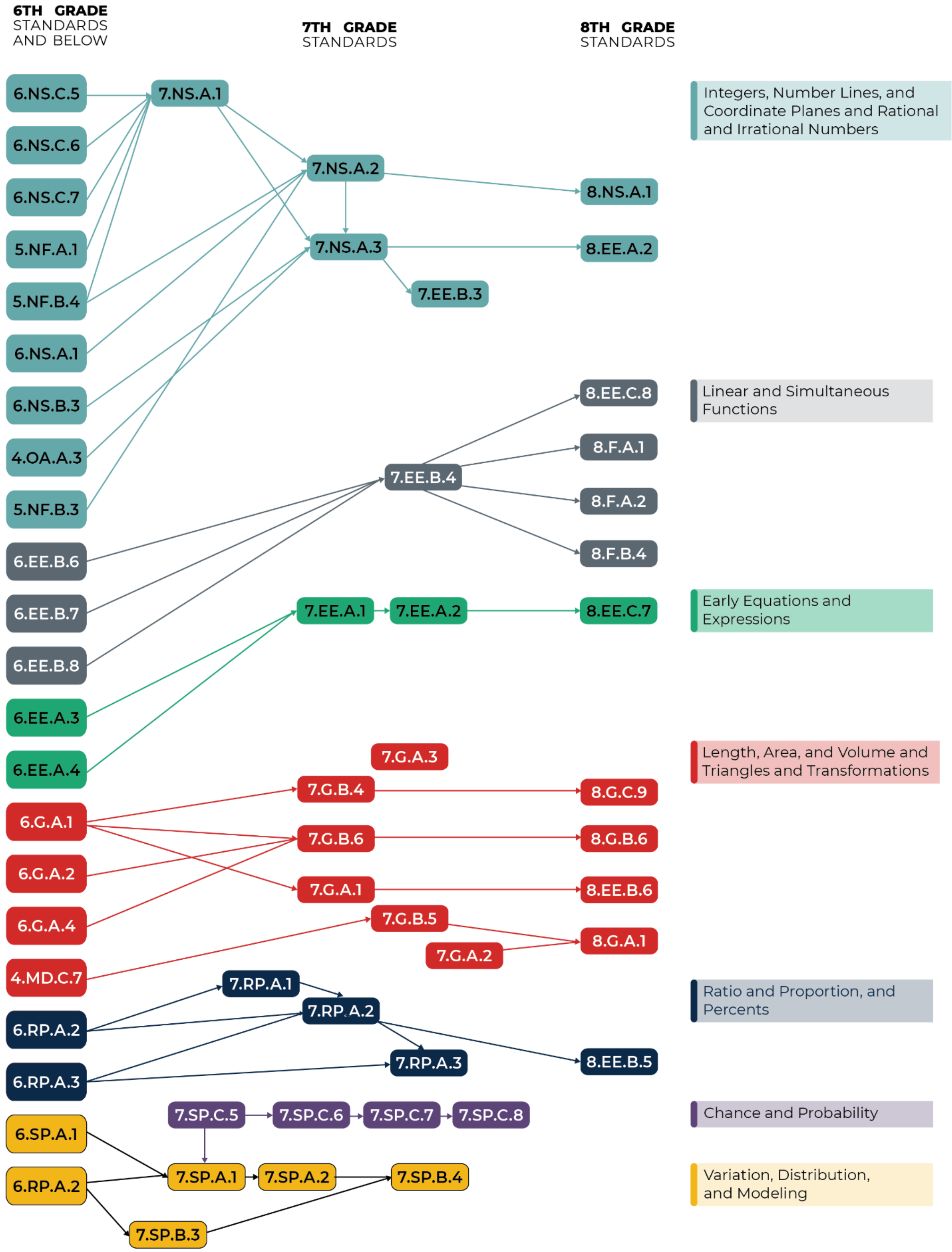
research-based learning progressions. Clements's and Sarama's (2011) learning progressions—or developmental progressions as they called them—are defined as "…levels of thinking, each more sophisticated than the last, through which children develop on their way to achieving the math goal. The developmental progression describes a typical path that children follow in developing an understanding and skill about that math topic" (Clements & Sarama, 2021, p. 4). Clements's and Sarama's developmental progressions were used as a foundation for the CCSSM as an attempt to weave together coherent progressions—rather than just assemble math topics—to help students learn mathematics at a deeper level (Zimba, 2013).

Once developed, researchers like Maloney (2013) and Confrey (2019) identified content-focused mathematical learning progressions within the CCSSM, based on research on children's cognitive development and the structure of mathematics (Maloney, 2013; Student Achievement Partners, 2013), to describe how standards connect across and within each grade level. Their mathematical learning progressions, as representations of the CCSSM, provide coherence among the standards and support teachers in planning and delivering instruction across grade levels (Confrey & Maloney, 2014). This effort has been key to creating higher-quality curricula aligned with the CCSSM that influence student learning (EdReports, 2023).

We define **mathematical learning progressions** as the clusters of knowledge that identify pathways students follow as they develop more sophisticated ways of reasoning about content in mathematical domains within and spanning across grade bands (Achieve the Core, n.d.; Zimba, 2013). Our definition stems from those above and Zimba's "coherent bodies of knowledge," or "… [how] standards progress from each other, coordinate with each other and most importantly cluster together into coherent bodies of knowledge" (p. 22).

In this study, we consider whether teachers' adaptations align with, or deviate from, the 18 mathematical learning progressions identified within the CCSS (Maloney, 2013; Student Achievement Partners, 2013). We can visualize mathematical learning progressions using the Coherence Map (Achieve the Core, n.d.), which is based on Dr. Jason Zimba's Graph of the Content Standards (Zimba, 2013). In Exhibit 1, we provide an example aligned with 7th grade, showing how standards connect within and across grades following the mathematical learning progressions. Each color represents a mathematical learning progression. For example, the green boxes and arrows represent the middle school standards within the *Early Equations and Expressions* learning progression (Maloney, 2013).

Exhibit 1. Seventh-grade mathematical learning progressions



Adapted from the Coherence Map (Achieve the Core, n.d.), Graph of the Content Standards (Zimba, 2013), and Maloney (2013).

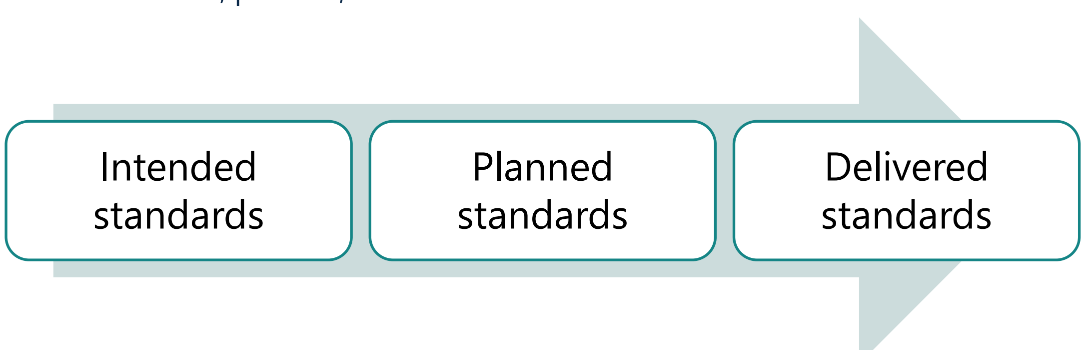
Adaptations from intended to planned to delivered content standards

Research has shown that teachers do adapt their curriculum, primarily by supplementing, adding, or replacing certain components to meet students' needs (Drake & Sherin, 2009; Hayden et al., 2023). When surveyed for the AMS project, most teachers indicated they make adaptations to at least half of their lessons each week by either augmenting existing content or changing the way it is delivered to meet the students' diverse learning needs.

When teachers choose to make adaptations to their curriculum, they may not know how to maintain alignment with mathematical learning progressions within each lesson (Boesen et. al, 2014). Instead, teachers may adapt lessons, or specific tasks within a lesson, to meet their students' needs, local or state mandates, or—based on their preferences without considering the standards—curriculum aims and objectives (Choppin et al., 2020; Cirillo et al., 2009). These adaptations may be based on their experiences with mathematics, early memories of learning mathematics, their current self-perceptions as mathematics learners, or their mathematical knowledge (Drake & Sherin, 2006).

There is limited evidence to inform whether teachers consider mathematical learning progressions when they choose to make adaptations (Hess, 2012). To understand this, we need to know several things: which standards curriculum developers intend for the materials in a lesson to address, which standards teachers plan to address in a lesson, and which standards teachers actually address while they deliver a lesson. We define (1) **intended standards** for a specific lesson as the standards identified in the materials districts purchase from product developers (Hill et al., 2008); (2) **planned standards** as those the teacher plans to focus on as they prepare the lesson (Turner et al., 2019); and (3) **delivered standards** as those observed while the teacher delivers their lesson (Drake & Sherin, 2060). These definitions stem from the AMS project's theory of action and are visualized in Exhibit 2. Because a teacher's adaptations in a lesson will have an impact on students' experiences and achievement (Konstantinou-Katzi, 2012) and because mathematical learning progressions are based on research on children's cognitive development and the structure of mathematics (Student Achievement Partners, 2013), the field would benefit from understanding the extent to which teachers' adaptations align with mathematical learning progressions in the CCSSM.

Exhibit 2. Intended, planned, and delivered standards



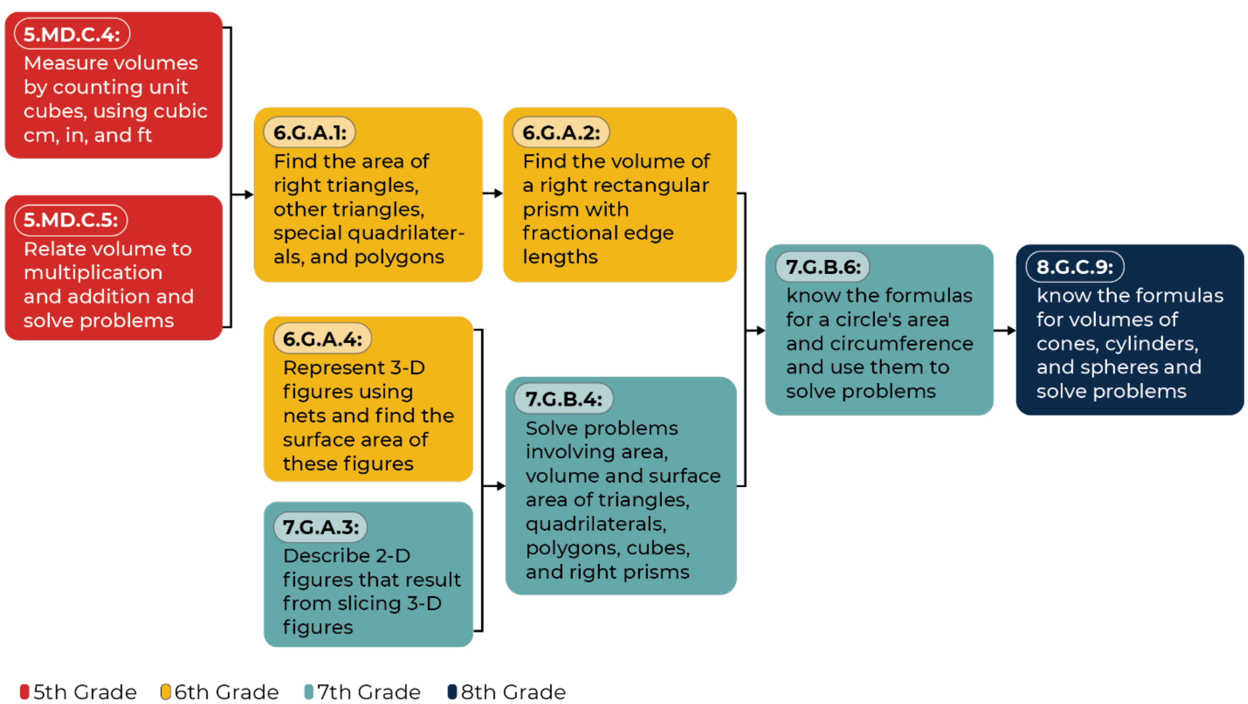
Productive and nonproductive adaptations

In Exhibit 3, we provide an example of the *Length, Area, and Volume* learning progression (Maloney, 2013). Red boxes indicate 5th-grade standards, yellow boxes indicate 6th-grade standards, teal boxes indicate 7th-grade standards, and blue boxes indicate 8th-grade standards. Arrows indicate prerequisite content standards within the learning progression—that is, standards students need to demonstrate before progressing to the subsequent standard. For example, students need to know the formula for a circle's area (7.G.B.6) before knowing the formula for the volume of a cone (8.G.C.9).

When a teacher makes a **productive adaptation**, they adjust the content of a lesson while maintaining alignment along the mathematical learning progressions the standards are built upon. Using Exhibit 3 as an illustration, a teacher may plan to cover 7.G.B.4 during the lesson but may adapt the content to address 6.G.A.2 while delivering the lesson because their students have not mastered that prerequisite content yet. This adaptation would be productive because it follows the mathematical learning progression.

However, if a teacher adjusts the content of a lesson such that they deviate from a mathematical learning progression, we would say the teacher made a **nonproductive adaptation**. In Exhibit 3, if a teacher planned to cover 7.G.B.6 but adjusted during the lesson to cover standard 7.EE.A.2, rewriting expressions to understand how quantities are related, we would say the teacher made a nonproductive adaptation because 7.EE.A.2 is not on the *Length, Area, and Volume* learning progression.

Exhibit 3. Length, area, and volume learning progression



We know that mathematical learning progressions are evidence-based pathways that build student's understanding of math topics (Maloney 2013; Confrey, 2019). We have a concern that as teachers make adaptations to their lessons, they do not consider mathematical learning progressions and make adaptations that move students away from the intended mathematical goal. Thus, it is imperative to understand whether teachers consider the content standards and mathematical learning progressions when making adaptations. Moreover, how and why teachers decide to adapt their lessons has implications for both curriculum design and professional learning.

In this study, we explore the following research questions:

1. Do teachers make productive adaptations to their lessons?
2. How often are teachers' adaptations below, on, or above grade level?
3. How do teachers describe their adaptations and their motivation for doing so?

Methods

Data sources for this report include (1) audio- or video-recorded classroom observations, (2) teacher interviews, (3) lesson plans and instructional materials provided by teachers, and (4) published lessons from six middle school math curricula. In this section, we describe these data sources and the methods we used to analyze them.

Data sources

Our study's sample included 37 teachers from 14 middle schools in four geographically dispersed urban school districts. For each teacher, we audio- or video-recorded one or two lessons during each of the two study years, resulting in 85 classroom observations. We also interviewed teachers after one of their observed lessons each year to understand how and why they made adaptations. The interviews also included questions designed to capture teachers' thinking beyond the lesson observed (for example, about culturally responsive teaching, their curriculum, and so on). In total, we conducted 46 interviews—16 teachers only in Year 1, 12 teachers only in Year 2, and 9 teachers in both Year 1 and Year 2.

As part of the study, teachers also provided the lesson plan and instructional materials associated with the classroom lesson we observed. We were unable to collect these for 17 observed lessons, resulting in a sample of lesson plans and instructional materials from 68 observed lessons. Finally, we identified the associated curriculum materials for each classroom observation, resulting in 85 published lessons across the six study curricula.

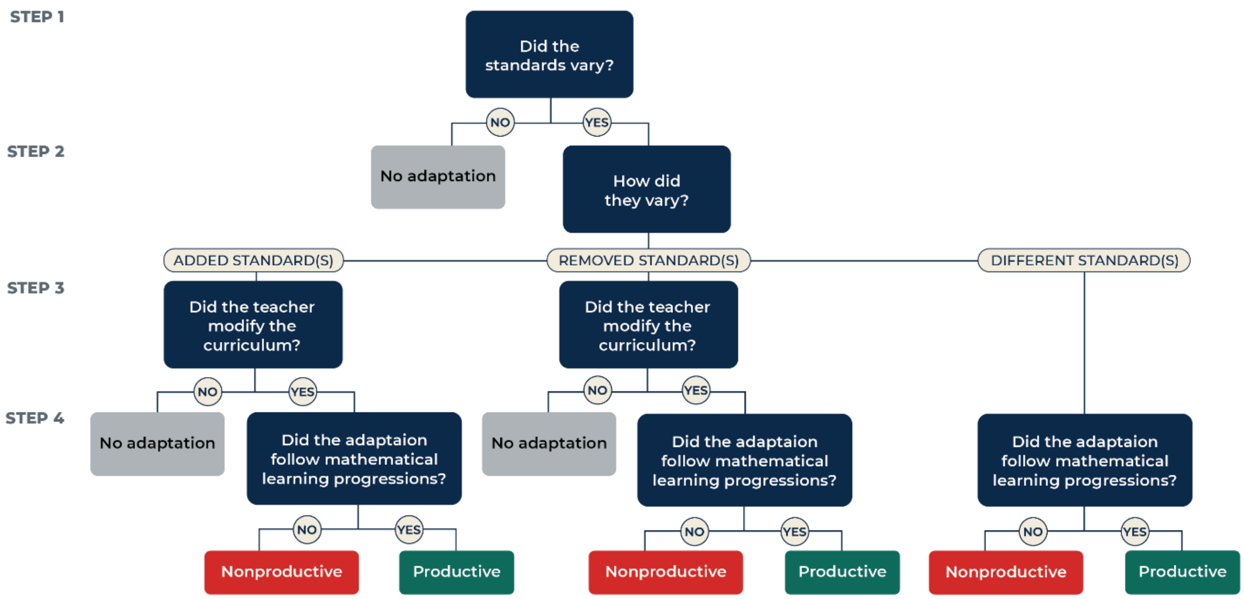
Identifying standards

To identify intended, planned, and delivered standards, we reviewed, respectively, the published curriculum materials identified as the basis for the lesson; the associated lesson plans and instructional materials teachers provided; and each classroom observation. For the intended standards, a coder reviewed the associated published curriculum for each classroom observation (as named by teachers in their interviews) and identified the content standards listed in the materials for each observed lesson. For the planned standards, a coder reviewed the teacher-provided lesson plans and instructional materials to identify the content standards the teacher listed in these materials. If the teacher did not list the content standards, two coders reviewed the materials and agreed on the planned standard. Finally, to identify the delivered standards, two coders watched or listened to each classroom observation to identify the content standard delivered during the lesson. For any observation where these two coders did not agree on a coded standard, a third coder reviewed it and the three came to a consensus[2].

Methods for research question 1: Do teachers make productive adaptations to their lessons?

To answer our first research question, we conducted a descriptive analysis to examine whether there was any variation between the standards and whether the variation was aligned with a mathematical learning progression. We followed the process described below and visually depicted in Exhibit 4.

Exhibit 4. Identifying how a lesson was adapted



- **Step 1: Did the standards vary?** If there was no variation between the intended and planned standards, the planned and delivered standards, or the intended and delivered standards, we identified the lesson as not being adapted. If there was a variation, we moved to Step 2.
- **Step 2: If the standards did vary, how did they vary?** That is, was a standard added or removed, or were there different standards? If a standard was added or removed, we moved to Step 3, and if there were different standards, we would move to Step 4.
- **Step 3: Did the teacher modify the curriculum?** That is, did the teacher alter the content of the curriculum in some way that resulted in different mathematical tasks, questions, or problems?[3] If not, we would identify the curriculum as not being adapted. If yes, we would move to Step 4.

[2] We identified elementary and middle school content standards covering numbers and operations—fractions; numbers and operations—in base ten; operations and algebraic thinking; ratios and proportional relationships; the number system; expressions and equations; geometry; and statistics and probability.

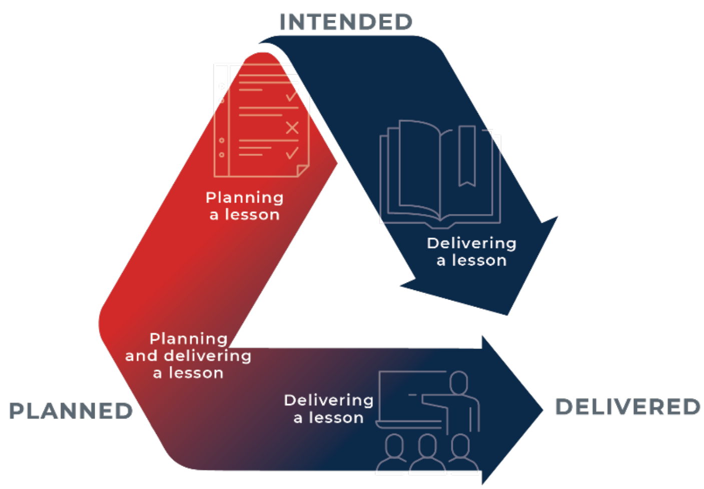
[3] For example, the curriculum identified two standards whereas the teacher only identified one in their planning even though their lesson plan directly aligned with the curriculum. In delivering the lesson, the teacher followed the curriculum as it was intended, so we identified the lesson as not being adapted. This occurred for five lessons. In two other lessons, the teacher either removed or replaced a problem such that a standard was removed.

- **Step 4: Did the adaptation follow mathematical learning progressions?**

To identify whether variations in standards followed mathematical learning progressions, we assessed whether standards were connected along the pathways identified in the Coherence Map (Achieve the Core, n.d.; Zimba, 2013). If they were connected, we identified the adaptation as productive; if they were not connected, we identified the adaptation as nonproductive. This resulted in identifying if a lesson was adapted and if so, whether the adaptation was productive or nonproductive for each of our 85 classroom observations.

When teachers make adaptations, they choose to do so either while planning or delivering a lesson. Exhibit 5 depicts how we identified whether teachers adapted their lesson while planning or delivering a lesson. We identified lessons teachers adapted while **planning** when there was a difference between the intended and planned standards. We identified lessons teachers adapted while **delivering** a lesson when there was a difference between the planned and delivered standards or when the teacher did not provide a lesson plan.[4], between the intended and delivered standard(s). We identified lessons teachers adapted while planning and delivering (that is, **both**) when there was a difference between the intended and planned standards as well as between the planned and delivered standards.[5]

Exhibit 5. Identifying when teachers made adaptations to lessons



Methods for research question 2: How often are teacher's adaptations below, on, or above grade level?

In answering our second research question, we only considered lessons in which teachers made adaptations. For these lessons, we identified each standard's grade level to understand if the variation in the standards was to higher, lower, or the same grade-level content. To exemplify this, if the intended standard was 7.NS.A.2 and the delivered standard was 6.NS.A.1, the adaptation would be to a lower grade

[4] Seven productive and four nonproductive adaptations were coded as occurring while teachers delivered a lesson because the teachers did not provide lesson plans. Thus, we could not assess if adaptations were made during the planning process.

[5] In our sample, teachers adapted three lessons while both planning and delivering a lesson (two of the adaptations were productive and one was nonproductive). In each of these lessons, the differences between the intended and planned and planned and enacted standards were either both productive or both nonproductive.

level; if the intended standard was 7.NS.A.2 and the delivered standard was 7.NS.A.1, the adaptation would be to the same grade level; and, if the intended standard was 7.NS.A.1 and the delivered standard was 8.NS.A.1, the adaptation would be to a higher grade level.

Methods for research question 3: How do teachers describe their adaptations and their motivation for doing so?

To answer our third research question, we analyzed 21 interviews from the subset of teachers who made both productive and nonproductive adaptations to their lessons. We conducted a qualitative analysis using a priori coding to identify **how** and **why** teachers made adaptations. In Exhibit 6, we list the a priori codes for how teachers made adaptations with example quotes from the teacher interviews. In Exhibit 7, we list the a priori codes for why teachers made adaptations with example quotes from the teacher interviews.

Exhibit 6. Descriptions and quotes of the a priori codes for how teachers made adaptations

Code	Description	Quote
Remove content	Removing mathematical tasks, questions, or problems	"I **took out some of the questions** that could have tricked them, like fractions or negatives."
Substitute content	Replacing mathematical tasks, questions, or problems with other tasks, questions, or problems from another resource	"Illustrative [Math] doesn't have a lot of practice, I **referred back to our old curriculum,** which was Big Ideas Math and **took a lot of problems from there**."
Alter content	Changing the numbers or context for mathematical tasks, questions, or problems in the curriculum	"I would **create my own warmups or cooldowns**. Like today, for the cooldown, I went ahead and I did a Kahoot instead of the cooldown they had in the Illustrative Math textbook."
Change sequence	Changing the order of content, mathematical tasks, questions, or problems in the curriculum	"In chapter seven, [were supposed to cover] squares and square roots and then cube and cube roots and then the Pythagorean theorem. But I like to talk about the **geometry application first** because I feel that's what this particular class likes."
Change delivery	Changing how instruction is delivered, for example, by having students work in pairs or groups rather than independently	"I always do either **pairs or groups of four**. And the textbook tells me, "Oh, this part **you should do independently**", but I do [groups] consistently."
Change timeline	Changing the amount of time dedicated to mathematical tasks, questions, or problems	"I might **shorten** this part to make sure I have enough time with the next part that I know they might be struggling with, to balance it out. By the end of the unit, we're pretty much on the time frame."
Translation	Translating mathematical tasks, questions, or problems into another language	"As I walk around and I have my English learners, I do **translate**. Of course, I want them to be able to understand and participate."

Exhibit 7. Descriptions and quotes of the a priori codes for why teachers made adaptations

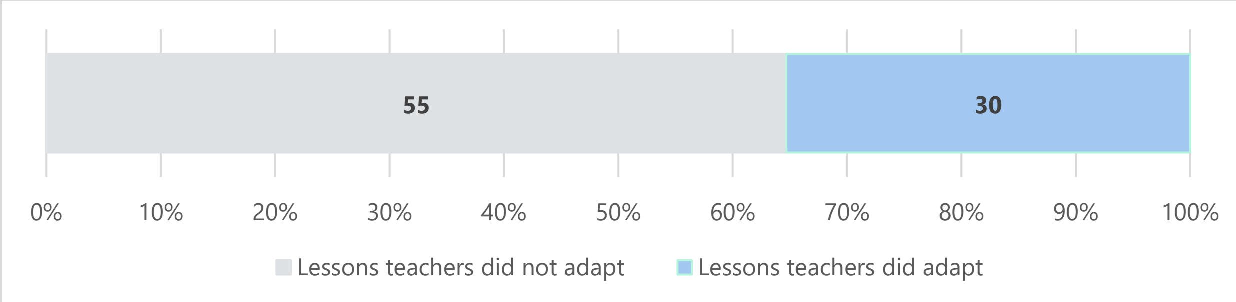
Code	Descriptions	Quote
Content	Better addressing math content	"They have an introduction to integers in sixth grade, but I don't know if it was done in sixth grade. It is supposed to be in the Illustrative Math curriculum and the assumption is they've already been introduced to it, but it's my first time **and I need to go over it again**."
Standards	Better addressing state standards	"I can't teach them how to divide using decimals —**which is what the goal is**—if they don't know how to do long division. So that's why I focused on problems they were supposed to learn in fifth grade, but most of them didn't. So that's something we were aware of."
Multilingual students	Making materials more appropriate for multilingual students	"I always try to tell them to **help [each other] out [because there's] only one of me. So, I asked them to translate**, and I try to use at least two or three different types of words like, there's so many different types of variations of Spanish, Caribbean, South American, [and] Spaniard."
IEPs	Making materials more appropriate for students with Individualized Education Programs (IEPs) or 504 plans	"I typically feel like I try to do that (**cater to students with IEPs**) as much as possible with every lesson."
Enrichment	Providing more enrichment activities for students who have already mastered the material	"The only thing I just wish I had harder problems for some students… So, I try to **give more real-world problems for our higher levels**."
Remediation	Providing more remediation activities for students who have not yet mastered the material	"We switched to small groups after a while. That way I can have some small groups moving forward while we work with the other groups so **they're getting the needed individual help to get them caught back up**."
Reduce time	Reducing the time it will take	"I **removed an activity that would have been four pages** instead of just an activity that would have taken a couple of minutes since it would have taken too long to get it done."

Results

Given our concern—that teachers making adaptations may not consider mathematical learning progressions and make adaptations that move students away from the intended mathematical goal—we first looked at how many observed lessons included adaptations. We found that in most observed lessons, the teacher did not make an adaptation. Teachers made adaptations to 30 of the lessons (Exhibit 8)[6].

[6] The AMS project also includes comparisons between standards-aligned curricula (rated green by EdReports) and curricula that have less alignment with standards (rated non-green). When examining the data for our study, we found no differences between teachers using green and non-green curricula. This is unsurprising because our analyses focused on the lesson level: that is, we considered individual lessons and the standards they are aligned with rather than the curriculum as a whole. In general, the standards alignment of an individual lesson is not dependent on whether the curriculum as a whole is standards aligned.

Exhibit 8. Are teachers making adaptations to their lessons?

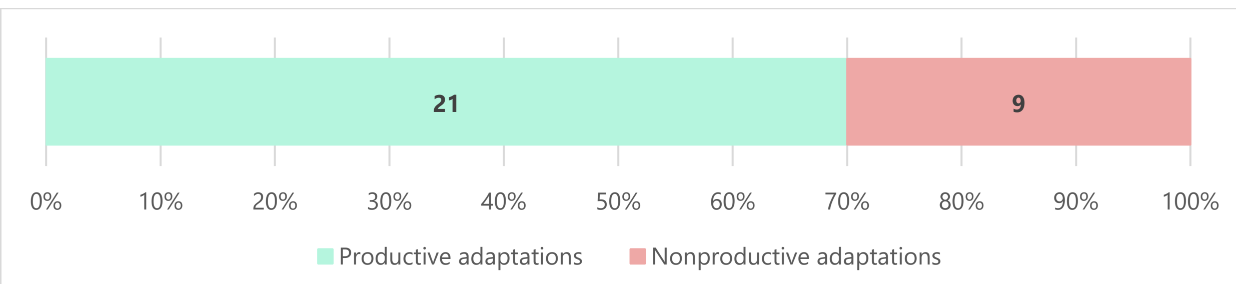


Turning to our research questions, we focus on a subset of 30 lessons from teachers who made adaptations. For this subset, we sought to explore whether teachers made adaptations that moved students away from the intended mathematical goal.

Do teachers make productive adaptations to their lessons? If so, how often are these to below-, on-, or above-grade-level standards?

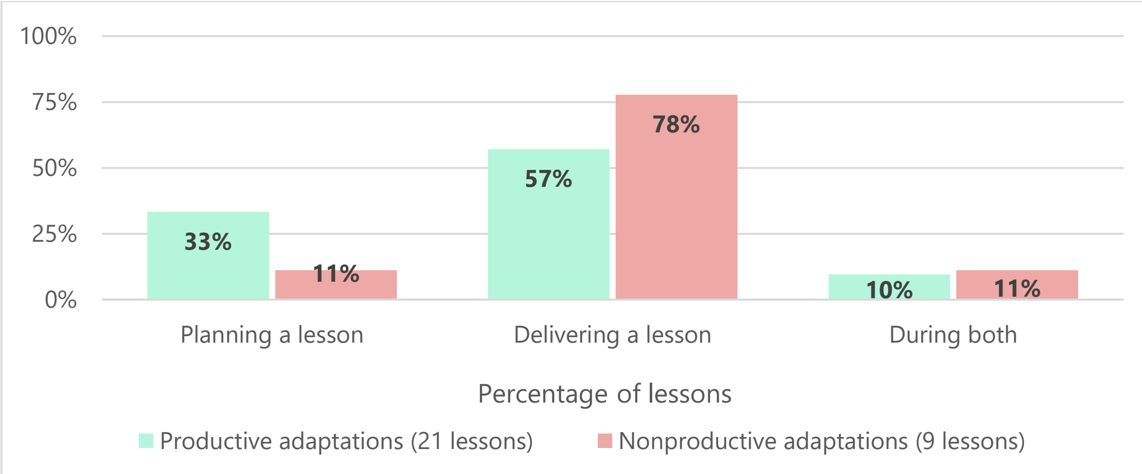
When teachers made adaptations to their lessons, they were more likely to make productive adaptations rather than nonproductive ones. Out of the 30 lessons where teachers chose to adapt their lessons, the overwhelming majority (70 percent) were productive and just 30 percent were nonproductive (Exhibit 9).

Exhibit 9. To what extent are teachers' adaptations to their lessons productive?



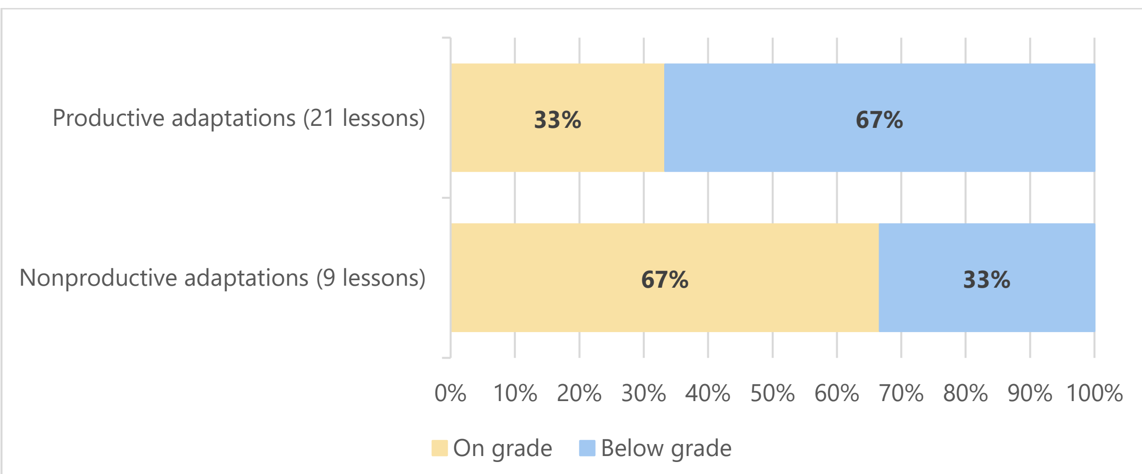
Teachers were more likely to make productive adaptations while planning a lesson and more likely to make nonproductive adaptations while delivering a lesson. Among teachers who make productive adaptations, 33 percent did so while planning the lesson and 57 percent did so while delivering a lesson. Conversely, among teachers who make nonproductive adaptations, 11 percent did so while planning the lesson and 78 percent did so while delivering a lesson (Exhibit 10).

Exhibit 10. When are teachers adapting math lessons?



Teachers' productive adaptations were more likely to be to below-grade-level content while teachers' nonproductive adaptations were more likely to be on grade level. Among teachers who made productive adaptations, 67 percent adapted to below-grade-level content and 33 percent to on-grade-level content. Conversely, among teachers who made nonproductive adaptations, 33 percent adapted to below-grade-level content and 67 percent to on-grade-level content (Exhibit 11). No adaptations were made to above-grade-level content.

Exhibit 11. How often are productive and nonproductive lesson adaptations made to below-, on-, or above-grade-level content?



Note: None of the adapted lessons we observed in our study were to above-grade-level content.

In Exhibits 12–19, we use the following color schemes: green shading indicates a productive adaptation, red shading indicates a nonproductive adaptation, blue shading indicates a below-grade-level adaptation, and yellow shading indicates an on-grade-level adaptation.

Comparing productive and nonproductive below-grade-level adaptations

In Exhibit 12, we provide two examples of productive below-grade-level adaptations while in Exhibit 13, we provide an example of a nonproductive below-grade-level adaptation.

Exhibit 12. Examples of productive below-grade-level adaptations

Productive, **below grade-level adaptations** generally resulted in lessons that focused on a prerequisite concept that should be mastered before the intended topic.

Productive, below grade-level	
Example In 7th grade *Illustrative Math*, an intended standard is to apply and extend previous understandings of addition and subtraction to add and subtract rational numbers **(7.NS.A.1)[a].** The teacher adapted the lesson by focusing on ordering rational numbers and understanding absolute values **(6.NS.C.7)[a]**.	**Why is it productive?** These standards both fall within the *Rational and Irrational Numbers* learning progression.[b]

Productive, below grade-level	
Example In 8th grade *Eureka Math*, an intended standard is to graph proportional relationships interpreting the unit rate as the slope of a graph **(8.EE.B.5)[a]**. The teacher adapted the lesson, focusing on understanding proportional relationships between quantities **(7.RP.A.2)[a]** which is prerequisite to interpreting the unit rate as the slope.	**Why is it productive?** These standards both fall within the *Rational and Irrational Numbers* learning progression.[b]

[a] Achieve the Core, n.d.
[b] Maloney, 2013

Exhibit 13. Example of a nonproductive below-grade-level adaptation

Nonproductive, below grade-level adaptations resulted in lessons that covered elementary content that did not align with middle school content.

Nonproductive, below grade-level	
Example In 6th grade Into Math, an intended standard is to draw polygons, find side lengths, and solve real-world math problems **(6.G.A.3)[a].** While enacting the lesson, the teacher had students find logos (such as car or shoe logos) and describe the characteristics of the geometric shapes in the logo which aligns with understanding shapes and their attributes **(3.G.A.1)[a]**.	**Why is it nonproductive?** **6.G.A.3** falls on the *Length, Area, and Volume* learning progression.[b] **3.G.A.1** falls on the *Shapes and Angles* learning progression.[b]

[a] Achieve the Core, n.d.
[b] Maloney, 2013

Comparing productive and nonproductive on-grade-level adaptations

In Exhibit 14, we provide an example of a productive on-grade-level adaptation while in Exhibit 15, we provide an example of nonproductive on-grade-level adaptation.

Exhibit 14. Example of a productive on-grade-level adaptation

Productive, **on-grade-level adaptations** generally emphasized grade-level content that was foundational to meeting the intended standard.

Productive, on grade-level	
Example A teacher adapted the 8th grade KEMS standard that has students analyze and solve pairs of simultaneous linear equations **(8.EE.C.8.)[a]** by focusing a majority of the lesson on graphing a line using the slope and y-intercept **(8.EE.B.6)[a]**.	**Why is it productive?** These standards both fall within the *Rational and Irrational Numbers* learning progression.[b]

[a] Achieve the Core, n.d.
[b] Maloney, 2013

Exhibit 15. Example of a nonproductive on-grade-level adaptation

Nonproductive, **on-grade-level adaptations** were generally instances where teachers deviated from the curriculum and the content in the lesson did not cover the intended standard.

Nonproductive, on grade-level	
Example	**Why is it nonproductive?**
In 8th grade *Eureka Math*, an intended standard is to estimate where square roots fall on a number line and the meaning of square roots **(8.NS.A.1** & **8.NS.A.2)**[a]. However, the teacher asked students to solve equations with square roots using a calculator **(8.EE.A.2)**[a].	**8.NS.A.1** & **8.NS.A.2** fall on the *Rational and Irrational Numbers* learning progression.[b] **8.EE.A.2** falls on the *Linear and Simultaneous Functions* learning progression.[b]

[a] Achieve the Core, n.d.
[b] Maloney, 2013

How do teachers describe their adaptations and their motivation for doing so?

Although the data we used to answer our first two research questions were at the lesson level, we used teacher-level data (that is, teacher interviews) to understand how and why teachers adapted their lessons[7]. For this, we focused only on the 49 percent of teachers (18 of 37) who made adaptations to their lessons. Of these, 13 teachers made productive adaptations to at least one lesson and indicated that they did consider the standards or mathematical learning progressions when thinking about how to better address their students' needs. We also found that the seven teachers who made nonproductive adaptations to at least one lesson did not indicate that they considered the standards or learning progressions when choosing to adapt their lessons[8].

Teachers who made productive adaptations more often indicated they considered the standards or mathematical learning progressions. These teachers indicated they changed the way content was delivered or altered the lesson in some way to better meet the needs of their students. In Exhibit 16, we provide examples of when teachers altered the content of the lesson or changed the way it was delivered to better address the math content their students needed to learn.

[7] Teachers were only interviewed once per year even if we observed more than one lesson.

[8] Two teachers made productive adaptations to at least one lesson and nonproductive adaptations to at least one other lesson. We found that these teachers did not consider the standards or learning progressions when choosing to adapt their lessons.

Exhibit 16. Examples of altering content or changing delivery to meet students' needs

Why is the lesson productive?

7.RP.A.2, **7.RP.A.3**, and **8.EE.B.5** all fall on the *Ratios and Proportion*, and Percents learning progression.[a]

(Intended and planned standard: 8.EE.B.5)

"It takes them a long time to come up with their simple solutions, especially because it's actually on a sixth-grade level. The first thing I did was say, 'I'm going to put simple solutions off to the side and we're just going to focus on working out the equations, solving for the variables and using our rules.'"

(Delivered standards: 7.RP.A.2, 7.RP.A.3)

Why is the lesson productive?

6.NS.B.2 and **5.NBT.B.6** fall on the *Division and Multiplication* learning progression.[a]

(Intended and planned standard: 6.NS.B.2, n.a.[b])

"I can't teach them how to divide using decimals—which is what the goal is—if they don't know how to do long division. So that's why I focused on problems they were supposed to learn in fifth grade, but most of them didn't. So that's something we were aware of."

(Delivered standards: 5.NBT.B.6)

Why is the lesson productive?

7.NS.A.1, **7.NS.A.2**, and **6.NS.C.5** all fall on the *Integers, Number Lines, and Coordinate Planes* learning progression.[a]

(Intended standard: 7.NS.A.1, 7.NS.A.2)

"This was the introductory lesson [and] my first time giving them integers... They have an introduction to integers in sixth grade, but I don't know if it was done in sixth grade. It is supposed to be in the Illustrative Math curriculum and the assumption is they've already been introduced to it, but it's my first time and I need to go over it again."

(Planned and delivered standards: 6.NS.C5)

[a] Maloney, 2013
[b] The teacher did not provide a lesson plan; thus, we were unable to code the planned standard.

Teachers who made nonproductive adaptations did not indicate they considered the standards or learning progressions when choosing to adapt their lessons. These teachers were more likely to indicate they changed the sequence of the content or altered the lesson in some way to reduce the time it would take to cover particular math content and to meet the needs of students performing below grade level, but when they did so, their adaptations did not follow mathematical learning progressions.

In Exhibit 17, we provide examples of when teachers changed the sequence to meet students performing below grade level; in Exhibit 18, we provide examples of when teachers altered the content of the lesson to meet students performing below grade level; in Exhibit 19, we provide examples of when teachers removed content from the lesson to reduce the time it takes to cover the material. In each of these examples, the adaptations made did not follow mathematical learning progressions, suggesting that these teachers did not consider how the standards were connected while they made adaptations.

Exhibit 17. Examples of changing the sequence to meet students below grade level

Why is the lesson nonproductive?

8.EE.A.2 falls on the *Linear and Simultaneous Functions* learning progression.[a] **7.G.B.4** falls on the *Length, Area, and Volume* learning progression.[a]

(Intended and planned standard: 8.EE.A.2)

"In chapter seven, [we're supposed to cover] squares and square roots and then cube and cube roots and then the Pythagorean theorem. But I like to talk about the geometry application first because I feel that's what this particular class likes."

(Delivered standards: 7.G.B.4)

Why is the lesson nonproductive?

8.NS.A.2 falls on the *Rational and Irrational Numbers* learning progression.[a] **8.EE.A.2** falls on the *Linear and Simultaneous Functions* learning progression.[a]

(Intended standard: 8.NS.A.2)

"I changed the sequence a little knowing where [students] are weak [in order] to build them up."

(Planned and delivered standards: 8.EE.A.2)

[a] Maloney, 2013

Exhibit 18. Examples of altering content to meet students below grade level

Why is the lesson nonproductive?

6.G.A.3 falls on the *Length, Area, and Volume* learning progression.[a] **3.G.A.1** falls on the *Shapes and Angles* learning progression.[a]

(Intended and planned standard: 6.G.A.3, n.a.[b])

"I know that [the students] know what a rectangle is from third or fourth grade. But I wanted them to start looking at more of the characteristics and comparing them in real life."

(Delivered standards: 3.G.A.1)

Why is the lesson nonproductive?

8.EE.B.5 falls on the *Ratios and Proportions, and Percents* learning progression.[a] **8.EE.C.7** falls on the *Linear and Simultaneous Functions* learning progression.[a] **7.G.B.4** falls on the *Length, Area, and Volume* learning progression.[a]

(Intended and planned standard: 8.EE.B5)

"I'll change the problems with [harder] numbers because [students] have no idea until we can actually go over them."

(Delivered standards: 8.EE.B7, 7.G.B.4)

Why is the lesson nonproductive?

8.EE.B.5 falls on the *Ratios and Proportions, and Percents* learning progression.[a]
8.F.B.4 falls on the *Linear and Simultaneous Functions* learning progression.[a]

(Intended and planned standard: 8.EE.B.5, n.a.[b])

"Eureka has a lesson on proportions, but I'm going to show them a PowerPoint [from a different curriculum] that's a lot easier and doesn't require a lot of reading... I was not trained on [Eureka] and reverted back to my previous curriculum."

(Delivered standards: 8.F.B.4)

[a] Maloney, 2013
[b] The teacher did not provide a lesson plan; thus, we were unable to code the planned standard.

Exhibit 19. Examples of removing content to reduce the time it will take

Why is the lesson nonproductive?

6.RP.A.3 falls on the *Rational and Irrational Numbers* learning progression.[a] **6.NS.A.1**, **5.NF.B.5** and **5.NF.B.7** fall on the *Division and Multiplication* learning progression.[a]

(Intended and planned standard: 6.NS.A.1, 6.RP.A.3)

"So, for [one] example, students were stuck. [Illustrative Math] asked students to identify if the total was going to be a fraction of a whole or bigger than a whole. That question threw them off because they truly don't need to answer it. So that part, I removed."

(Delivered standards: 5.NF.B.5, 5.NF.B.7)

Why is the lesson nonproductive?

8.EE.B.5 falls on the *Ratios and Proportion, and Percents* learning progression.[a] **8.EE.C.7** falls on the *Linear and Simultaneous Functions* learning progression.[a]

(Intended and planned standard: 8.EE.B.5, 8.EE.C7)

"I didn't include the activity that would have asked us to [solve equations] because it would have been four pages of an activity that would have taken too long to get it done."

(Delivered standard: 8.F.A.1)

[a] Maloney, 2013

Discussion and Implications

Districts may further support teachers' adaptations by considering professional learning that emphasizes how content standards align within mathematical learning progressions and the planning needed to make adaptations productively. Although most lesson adaptations aligned with mathematical learning progressions, our findings show that some teachers may not consider the standards or mathematical learning progressions when making adaptations. Districts may want to support these teachers by considering professional learning that encourages teachers to reflect on a lesson's standards, the needs of their students, and ways to adapt lessons within mathematical learning progressions may help teachers make productive adaptations.

Moreover, our findings show that teachers were more likely to make nonproductive adaptations while delivering a lesson and these were more likely to be to on-grade-level content. In our sample, these nonproductive adaptations were more likely to be made by teachers with more experience teaching math, higher education, and more advanced certification, suggesting these teachers may have a preferred way to teach mathematics. Professional learning could emphasize how standards within the same grade level connect and which standards students need to master before progressing to others within the same grade level. For example, knowing how to recognize and represent proportional relationships between quantities (7.RP.A.2) is prerequisite to solving problems involving scale drawings of geometric figures (7.G.A.1). Professional learning, focused on how to interpret and enact curricular standards and objectives, will help teachers adapt instructional materials to keep students moving toward the intended mathematical goal.

Districts should consider adhering to high-quality, standards-aligned materials that emphasize mathematical learning progressions rather than adapting the curricula they purchase or creating their own. One of the districts in our study, and many teachers across districts, adapted or created their curricula, unit plans, or lessons. Research has demonstrated that when school staff attempt to develop curricular materials instead of relying on published materials, they often misinterpret the intent of the content standards they were built upon (Pak et al., 2020). Districts can use publicly available information, such as EdReports, to identify instructional materials.

Teachers should consider a variety of formative assessments (for example, multiple-choice, performance tasks, and project-based learning) to assess how students are making progress toward meeting standards and identify additional support needed to help students become proficient. Teachers should use various assessments to understand which standards students have mastered. In turn, this will help them identify what content students are ready to learn, align it with the mathematical learning progressions, and incorporate the content into their instructional pacing and materials. Teachers may need additional time in their pacing to prioritize foundational skills that build toward the intended standards. When additional time is not available, districts and teachers should consider providing additional support to students outside of general education classes. This may include just in time support, extended learning opportunities, or tutoring to help students meet proficiency without using too much general education class time.

Teachers should consider how and when to adapt lessons to include above-grade-level content. In our study sample, all adaptations made by teachers were to below- or on-grade-level standards. Teachers should reflect on how they might productively adapt lessons to include examples, problems, or tasks that introduce students to above-grade-level content, challenging students to higher levels of thinking instead of solely focusing on supporting unfinished learning. Using the learning progressions to help identify what students have mastered and appropriate next steps in their learning toward the intended mathematical goal is imperative. Teachers may need to provide additional scaffolding or support when introducing students to this content, but this will allow students to become accustomed to math they might see in future grades.

Limitations

This study focuses specifically on content adaptations—as evidenced in the CCSSM—rather than adaptations serving other purposes, such as adapting lessons to be more culturally responsive or translating content for English learners. Other studies within the AMS project will explore how non-content adaptations relate to the enactment of middle school mathematics curricula.

Our sample size (37 teachers and 85 classroom observations) is particularly small and may not be generalizable. Within this sample, we only observed one or two lessons from each teacher each year, so we may not have the necessary context to determine how these lessons fit within the longer-term pacing and plan for classroom instruction as well as how consistently teachers make adaptations to lessons. Moreover, we did not collect formative or summative student assessment data, so we could not determine if students needed the adaptations teachers were making nor whether they were beneficial in supporting student learning. While we coded whether the adapted standards were below, on, or above grade level, we could not determine if the students in the class needed those specific adaptations.

Conclusion and Future Research

Consistent with prior research (Drake & Sherin, 2009; Hayden et al., 2023; Konstantinou-Katzi, 2012), we found that teachers supplement or replace curriculum materials to better meet students' needs. Even though most adaptations made by teachers in our study were productive, some teachers still adapt their lessons in ways that do not align with mathematical learning progressions. There may be an opportunity for professional learning to support teachers in adapting lessons in ways that build students' understanding of math content and maintain alignment with mathematical learning progressions. We hypothesize that this will ensure students receive ambitious and inclusive instruction and have a better classroom experience in terms of their mathematics enjoyment, achievement identity, performance, persistence, self-efficacy, and growth mindset.

Below we describe what we think interested parties might do with the above findings and implications:

1. **Districts** may consider professional learning that emphasizes mathematical learning progressions.
2. **Teachers and math coaches** should reflect on the standards intended by the curriculum, the standards they plan to address, and the standards they deliver in the classroom, as well as on making productive adaptations.
3. **Curriculum developers** should more clearly lay out how a lesson's standards build from previous standards and how they build toward future standards, as well as how to adapt the material to those standards.
4. **Researchers** should prioritize efforts to understand if professional education on mathematical learning progressions helps teachers make productive adaptations and if it ultimately improves a student's classroom experience and learning. Researchers could connect teachers' adaptations to formative student assessment data to determine if adaptations are needed and whether productive adaptations improve student learning more so than nonproductive adaptations. There is also an opportunity to understand how teacher characteristics influence adaptation decisions.

The qualitative research we include in this report complements our educator practice guide that provides educators with exemplars and resources on how to make productive standard adaptations across multiple scenarios. For more information about the broader AMS project, please visit Middle School Math: Culturally Responsive Materials, Teacher Professional Learning, and Student Engagement.

References

Achieve the Core. (n.d.) *Coherence map*. Retrieved December 15th, 2023, from https://tools.achievethecore.org/coherence-map/

Agodini, R., & Harris, B. (2010). An experimental evaluation of four elementary school math curricula. *Journal of Research on Educational Effectiveness, 3*(3), 199–253.

Allensworth, E., Cashdollar, S., & Gwynne, J. (2021, February 7). Improvements in math instruction and student achievement through professional learning around the Common Core State Standards in Chicago. *AERA Open, 7*. https://doi.org/10.1177/2332858420986872

Battista, M. (2010, December). Engaging students in meaningful mathematics learning: Different perspectives, complementary goals. *Journal of Urban Mathematics Education, 3*(2). https://doi.org/10.21423/jume-v3i2a115

Bleiberg, J. (2021, April 26). Does the Common Core have a common effect? An exploration of effects on academically vulnerable students. *AERA Open*, *77*. https://doi.org/10.1177/23328584211010727

Boesen J., Helenius, O., Bergqvist, E., Bergqvist, T., Lithner, J., Palm, T., & Palmberg, B. (2014, March). Developing mathematical competence: From the intended to the enacted curriculum. *The Journal of Mathematica Behavior, 33*, 72–87. https://doi.org/10.1016/j.jmathb.2013.10.001

Brookings. (2021, March). *Why Common Core failed*. https://www.brookings.edu/articles/why-common-core-failed/

Choppin, J., McDuffie, A. R., Drake, C., & Davis. J. (2020, December 9). The role of instructional materials in the relationship between the official curriculum and the enacted curriculum. *Mathematical Thinking and Learning*, *24*(2), 123–148. https://doi.org/10.1080/10986065.2020.1855376

Cirillo, M., Drake, C., & Herbel-Eisenmann, B. (2009). Contemporary curriculum issues: Curriculum vision and coherence: Adapting curriculum to focus on authentic mathematics. *The Mathematics Teacher, 103*(1), 70–75. https://doi.org/10.5951/MT.103.1.0070

Clements, D. H., Sarama, J., Spitler, M. E., Lange, A. A., & Wolfe, C. B. (2011, March). Mathematics learned by young children in an intervention based on learning trajectories: A large-scale cluster randomized trial. *Journal for Research in Mathematics Education, 42*(2), 127–166. 10.5951/jresematheduc.42.2.0127

Clements, D. H., & Sarama., J. (2021). *Learning and teaching early math: The learning trajectories approach*. Routledge.

Cobb, P., & Jackson., K. J. (2011). Towards an empirically grounded theory of action for improving the quality of mathematics teaching at scale. *Mathematics Teacher Education and Development*, *13*(1), 6–33.

Common Core State Standards Initiative. (n.d.) *Mathematics standards*. Retrieved on March 28, 2024, from https://corestandards.org/mathematics-standards/

Confrey, J., & Maloney, A. (2014). Linking standards and learning trajectories: Boundary objects and representations. In *Learning over time: Learning trajectories in mathematics education*. Information Age Publishing.

Confrey, J. (2019). Future of education and skills 2030: Curriculum analysis. *Organisation for Economic Co-Operation and Development*, 44(3). http://www.oecd.org/education/2030/A-Synthesis-of-Research-on-Learning-Trajectories-Progressions-in-Mathematics.pdf

Desimone, L. M., & Garet, M. S. (2015). Best practices in teachers' professional development in the United States. *Psychology, Society and Education, 7*(3), 252–263.

Doan, S., Kaufman, J. H., Woo, A., Prado Tuma, A., Diliberti, M. K., & Lee, S. (2022). *How states are creating conditions for use of high-quality instructional materials in K–12 classrooms.* RAND Corporation. https://www.rand.org/pubs/research_reports/RRA134-13.html

Drake, C., & Sherin, M. G. (2006). Practicing change: Curriculum adaptation and teacher narrative in the context of mathematics education reform. *Curriculum Inquiry*, 36, 153–187. https://doi.org/10.1111/j.1467-873X.2006.00351.x

Duncan, R. G. (2009, April). Learning progressions: Aligning curriculum, instruction, and assessment. *Journal of Research in Science Teaching*, *46*(6), 606–609.

EdReports. (2023, October). *State of the instructional materials market: Use of aligned materials in 2022*. https://www.edreports.org/resources/article/state-of-the-instructional-materials-market-use-of-aligned-materials-in-2022

Elmore, R. F., Forman, M. L., Stosich, E. L., & Bocala, C. (2014). *The internal coherence assessment protocol & developmental framework: Building the organizational capacity for instructional improvement in schools.* Strategic Education Research Partnership. https://files.eric.ed.gov/fulltext/ED564482.pdf

Hayden, S. M., Gubbins, E. J., Cody, R. A., & Boldt, G. T. (2023). Teachers' perceptions of differentiation following a math curriculum implementation study. *Journal for the Education of the Gifted, 47*(1). https://doi.org/10.1177/01623532231215092

Hess, K. K. (2012, January). *Learning progressions in K-8 classrooms: How progress maps can influence classroom practice and perceptions and help teachers make more informed instructional decisions in support of struggling learners (Synthesis Report 87).* National Center on Educational Outcomes, University of Minnesota.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, *39*(4), 372–400.

Hill, H. C., Lynch, K., Gonzalez, K. E., & Pollard, C. (2020, January 27). *Professional development that improves STEM outcomes.* Phi Delta Kappan.

Hume, A., & Coll, R. (2010, February). Authentic student inquiry: The mismatch between the intended curriculum and the student-experienced curriculum. *Research in Science & Technological Education*, 28(1), 43–62. https://doi.org/10.1080/02635140903513565

Konstantinou-Katzi, P., Tsolaki, E., Meletiou-Mavrotheris, M., & Koutselini, M. (2013). Differentiation of teaching and learning mathematics: An action research study in tertiary education. *International Journal of Mathematical Education in Science and Technology, 44*(3), 332–349. 10.1080/0020739X.2012.714491

Loveless, T. (2021, April). *Between the state and the school house: Understanding the failure of Common Core*. Harvard Education Press.

Makarin, A. (2018, August). Can online off-the-shelf lessons improve student outcomes? Evidence from a field experiment. *American Economic Journal: Economic Policy*, *10*(3), 226–254.

Maloney, A. (2013, October 18). *TurnOnCCMath.net: Learning trajectories to interpret the Common Core Standards.* Presentation at the NCTM Regional Conference, Baltimore, MD.

Moyer, J. C., Cai, J., Wang, N., & Nie, B. (2011). Impact of curriculum reform: Evidence of change in classroom practice in the United States. *International Journal of Educational Research, 50*(2), 87–99.

National Center for Education Evaluation and Regional Assistance. (2013, September). *After two years, three elementary math curricula outperform a fourth*. U.S. Department of Education, Institute of Education Sciences.

National Center for Education Statistics. (2021). *Total expenditures for public elementary and secondary education and other related programs, by function and subfunction: Selected years, 1990-91 through 2018-19*. Digest of Education Statistics. https://nces.ed.gov/programs/digest/d21/tables/dt21_236.20.asp

Pak, K., Polikoff, M. S., Desimone, L. M., & Garcia, E. S. (2020, June 12). The adaptive challenges of curriculum implementation: Insights for educational leaders driving standards-based reform. *AERA Open, 6*(2). https://doi.org/10.1177/2332858420932828

Parrish, C. W., & Bryd, K. O. (2022). Cognitively demanding tasks: Supporting students and teachers during engagement and implementation. *International Electronic Journal of Mathematics Education, 17*(1). https://doi.org/10.29333/iejme/11475

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. *Second Handbook of Research on Mathematics Teaching and Learning* (vol. 1, pp. 319–369). Information Age Publishing.

Sherin, M. G., & Drake, C. (2009). Curriculum strategy framework: Investigating patterns in teachers' use of a reform-based elementary mathematics curriculum. *Journal of Curriculum Studies*, *41*(4), 467–500. https://doi.org/10.1080/00220270802696115

Smith, C., Wiser, M., Anderson, C. W., & Krajik, J. (2006, January). Implications of research on children's learning for standards and assessment: A proposed learning progression for matter and the atomic-molecular theory. *Measurement: Interdisciplinary Research and Perspectives*, *4*(2), 1–98.

Student Achievement Partners. (2013, July). P*rogressions documents for the Common Core State Standards for Mathematics.* Achieve The Core. https://achievethecore.org/page/254/progressions-documents-for-the-common-core-state-standards-for-mathematics

Squires, D. (2012). Curriculum alignment research suggests that alignment can improve student achievement. *The Clearing House, 85*(4), 129–135. http://www.jstor.org/stable/23212870

Tarr, J. E., Reys, R. E., Reys, B. J. Chavez, O., Shih, J., & Osterlind, S. J. (2008, May). The impact of middle-grades mathematics curricula and the classroom learning environment on student achievement. *Journal for Research in Mathematics Education, 39*(3), 247–280.

Turner, E. E., Drake, C., McDuffie, A. R., Aguirre, J., Bartell, T. G., & Foote, M. Q. (2012). Promoting equity in mathematics teacher preparation: A framework for advancing teacher learning of children's multiple mathematics knowledge bases. *Journal of Mathematics Teacher Education, 15*(1), 67–82.

Turner, E., Roth McDuffie, A., Sugimoto, A., Aguirre, J., Bartell, T. G., Drake, C., & Witters, A. (2019). A study of early career teachers' practices related to language and language diversity during mathematics instruction. *Mathematical Thinking and Learning, 21*(1), 1–27.

Whitehurst, G. J. (2009). *Don't forget curriculum*. Brown Center Letters on Education, #3. Brookings. https://www.brookings.edu/articles/dont-forget-curriculum/

Zimba, J., McCallum, W., & Daro, P. (2013, October). *The structure is the standards.* Achieve the Core, Student Achievement Partners. https://achievethecore.org/page/845/the-structure-is-the-standards

Zimba, J. (2013, October). "A graph of the content standards." Achieve the Core, Student Achievement Partners, October 2013. https://achievethecore.org/page/844/a-graph-of-the-content-standards